AMRL-TR-79-77

LEVEL II

DEVELOPMENT OF OBSERVER MODEL FOR AAA TRACKER RESPONSE

ROBERT S. KOU
BETTY C. GLASS
SYSTEMS RESEARCH LABORATORIES, INC.
2800 INDIAN RIPPLE ROAD
DAYTON, OHIO 45440

AUGUST 1979

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AEROSPACE MEDICAL DIVISION
AIR FORCE SYSTEMS COMMAND
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AMRL-TR-79-77

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FOR THE COMMANDER

CHARLES BATES, JR.
Chief
Human Engineering Division
Air Force Aerospace Medical Research Laboratory

AIR FORCE/56780/17 January 1980 — 100

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AMRL-TR-79-77
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): DEVELOPMENT OF OBSERVER MODEL FOR AAA TRACKER RESPONSE
5. TYPE OF REPORT & PERIOD COVERED: Technical Report
6. PERFORMING ORG. REPORT NUMBER: 6872-4
7. AUTHOR(s): Robert S. Kou, Betty C. Glass
8. CONTRACT OR GRANT NUMBER(s): F33615-79-C-0500
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Systems Research Laboratories, Inc., 2800 Indian Ripple Road, Dayton, Ohio 45440
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 62202F, 6893-04-33
11. CONTROLLING OFFICE NAME AND ADDRESS: Aerospace Medical Research Laboratory, Aerospace Medical Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio 45433
12. REPORT DATE: August 1979
13. NUMBER OF PAGES: 74
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):
15. SECURITY CLASS. (of this report): Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:

16. DISTRIBUTION STATEMENT (of this Report): Approved for public release; distribution unlimited
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report):
18. SUPPLEMENTARY NOTES:
19. KEY WORDS (Continue on reverse side if necessary and identify by block number): Anti-aircraft Artillery System, Luenberger Observer Theory, Gunner Model, Parameter Identification, Simulation
20. ABSTRACT (Continue on reverse side if necessary and identify by block number):

This report describes the development of a mathematical model for gunner's tracking performance in MTQ Mode II tracking task. The Luenberger observer theory is used to design the gunner model which is composed of three elements--a reduced-order observer, a feedback controller, and a remnant element. An important feature of the model is that its structure is simple, hence the computer simulation of anti-aircraft artillery tracking systems using the gunner model requires only a short execution time. A parameter identification

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program based on the combined least squares curve-fitting method and the modified Gauss Newton gradient algorithm is developed to determine parameters of the model systematically. Model predictions of both azimuth and elevation tracking errors for several target flyby and maneuvering trajectories are shown to be in excellent agreement with the empirical data obtained from simulation experiments conducted at the Aerospace Medical Research Laboratory, Wright-Patterson AFB, Ohio. It is concluded that the antiaircraft gunner model based on the observer theory can be accurately and efficiently used to study AAA weapon effectiveness and aircraft survivability.

SUMMARY

This report describes the development of a mathematical model for gunner's tracking performance in MTQ Mode II tracking task which is a linear time-varying antiaircraft artillery system. The Luenberger observer theory is used to design the gunner model which is composed of three elements--a reduced-order observer, a feedback controller, and a remnant element. An important feature of the model is that its structure is simple, hence the computer simulation of man-in-the-loop AAA tracking systems using the gunner model requires only a short execution time. A parameter identification program based on the combined least squares curve-fitting method and the modified Gauss Newton gradient algorithm is developed to determine parameters of the model systematically. Model predictions of both azimuth and elevation tracking errors for several target flyby and manuevering trajectories are shown to be in excellent agreement with the empirical data obtained from manned AAA simulation experiments conducted at the Aerospace Medical Research Laboratory, Wright-Patterson AFB, Ohio. It is concluded that the antiaircraft gunner model based on the observer theory can be accurately and efficiently used to study AAA weapon effectiveness and aircraft survivability.

PREFACE

This report documents a study performed by Systems Research Laboratories, Inc. (SRL), Dayton, Ohio, for the Aerospace Medical Research Laboratory (AMRL), Manned-Systems Effectiveness Division, Manned Threat Quantification program. This work was performed under Contract F33615-76-C-5001. The Contract Monitor was Mr. Robert E. Van Patten and the Technical Manager was Capt. Jon Hull. The SRL Project Manager was Mr. Charles McKeag.

The authors wish to extend their deepest appreciation and gratitude to Dr. Carroll N. Day of Manned-Systems Effectiveness Division of the Aerospace Medical Research Laboratory, WPAFB, for stimulating the basic concept of this research and for many valuable discussions which made the successful development of the human operator model possible. The authors also wish to thank Mr. Maris M. Vikmanis (SRL), Dr. Daniel W. Repperger (AMRL) and Mr. Walter C. Summers for their constructive criticism and helpful comments.

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INTRODUCTION

The Manned-Systems Effectiveness Division of the Aerospace Medical Research Laboratory (AMRL/ME), WPAFB, Ohio, performs research on the gunner's tracking response in Anti-Aircraft Artillery (AAA) systems. To analyze the performance of an AAA system, a mathematical gunner model representing the control characteristics of the gunner in the compensating tracking task is required. Several human operator models have been developed:

1. McRuer Crossover Model [1] - based on classical control theory,
2. Optimal Control Model [2], [3], [4] - based on optimal control and estimation theory,
3. PID Structure Modified Optimal Control Model [5] - Simplified Optimal Control Model.

A brief comparison of these models can be found in Appendix A. Although most of these models can predict tracking errors, they either have a complicated model structure and/or it is difficult to determine values for the model parameters for a given weapon system.

We wish to design a model whose structure is simple, whose parameters could be identified systematically, whose computer implementation would be fast and efficient, and whose output would accurately describe the tracker's response characteristics.

This report presents an AAA gunner model based on the Luenberger observer theory [6], [7], [8]. The observer theory provides a new method to obtain an approximate estimation for the state of an observed system. The characteristics of an observer are somewhat free to the extent that they can be determined by the designer through the proper selection of an observer gain. There is no Riccati equation involved in the observer design. The simplicity of the observer design and its capability for state estimation make the observer theory an

attractive design method. Then the estimated state vector can be used to implement a linear state variable feedback controller which represents the gunner's tracking function. Furthermore, it is assumed that the effects of all randomness sources in the AAA closed loop system can be lumped into one remnant element. Therefore, the structure of this model is simple. In addition, a parameter identification program based on least squares curve-fitting method and the Gauss Newton algorithm [9] was developed for this model. Hence, its parameters can be easily determined. A computer simulation program OMS (Observer Model Simulation) of the AAA tracking task using this model was also developed. Simulation results showed that the model predictions of the tracking errors were in excellent agreement with the actual gunner response data of the manned AAA simulation conducted at the Aerospace Medical Research Laboratory, WPAFB, Ohio. The computer execution time of the AAA simulation using this simple model is very fast. The description of a AAA gun system and the design of the observer model are included in Section II. Section III describes the method to determine the model parameters. Discretization techniques for computer implementation will be included in Section IV along with the computer simulation results. The conclusion will be discussed in Section V.

SECTION II

MODEL DEVELOPMENT

A. Description of An AAA Gun System

The tracking task of an anti-aircraft artillery (AAA) gun system can be described by a closed loop block diagram as shown in Figure 1. Two gunners, one each for azimuth and elevation axes, play the role of controller in the man-machine feedback control system. From the visual display, each gunner observed the tracking error e_T, which is the difference between the target position angle θ_T, and the gunsight angle θ_g. Independently, the gunners operated hand cranks to control the gunsight system to align the gunsight angle (output) with the target position angle (input). Therefore, the azimuth tracking task is decoupled from the elevation tracking task in this AAA system. Four trajectories, Figure 2, of the target aircraft were used as input to the AAA system. These trajectories are deterministic functions of time, although the gunners do not know their dynamic properties, $\dot{\theta}_T$, $\ddot{\theta}_T$, etc. In order to develop a mathematical model of the gunner response characteristics in an AAA compensatory tracking task, we first need to describe the mathematical representations of the gunsight dynamics. Let us consider an AAA system with the following gunsight transfer function.

$$\frac{\theta_g(s)}{U(s)} = \frac{64\,(s+1)}{s(s^2 + 12s + 64)}$$

Based on frequency domain analysis of the inputs (target trajectories), it was found that the frequency bandwidths of all the trajectories in Figure 2 are around 0.2 Hz. Thus, a simplified gunsight transfer function

$$\frac{\theta_g(s)}{U(s)} \cong \frac{1}{s} \qquad (1)$$

can be used for the model design and simulation analysis.

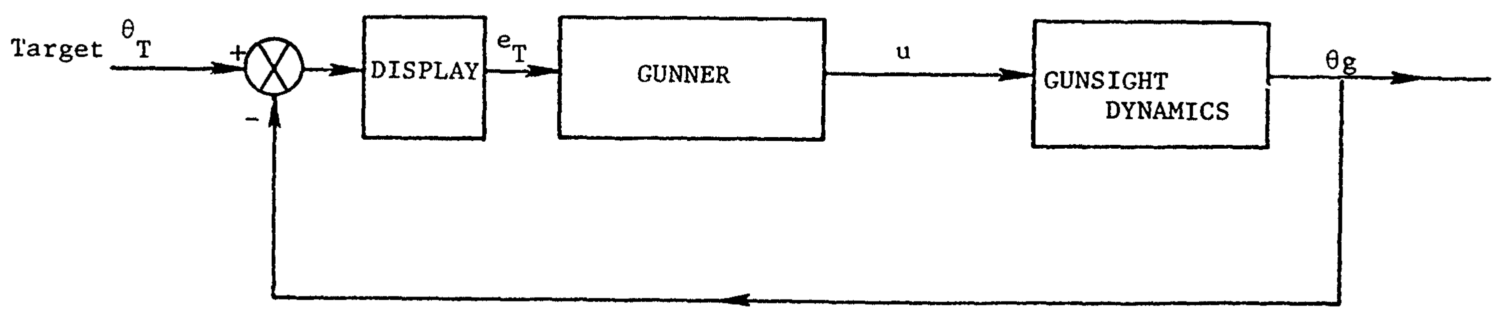
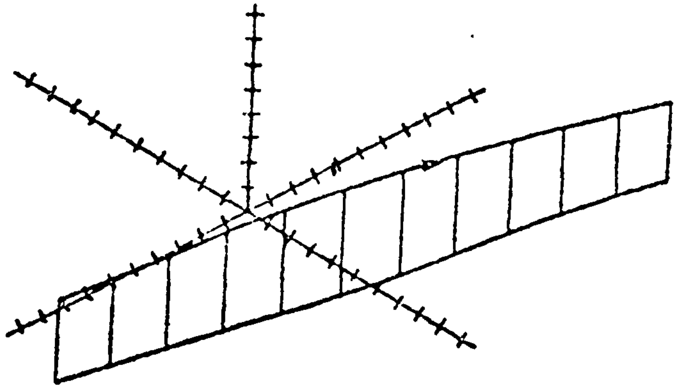


FIGURE 1: BLOCK DIAGRAM OF AN AAA CLOSED LOOP SYSTEM

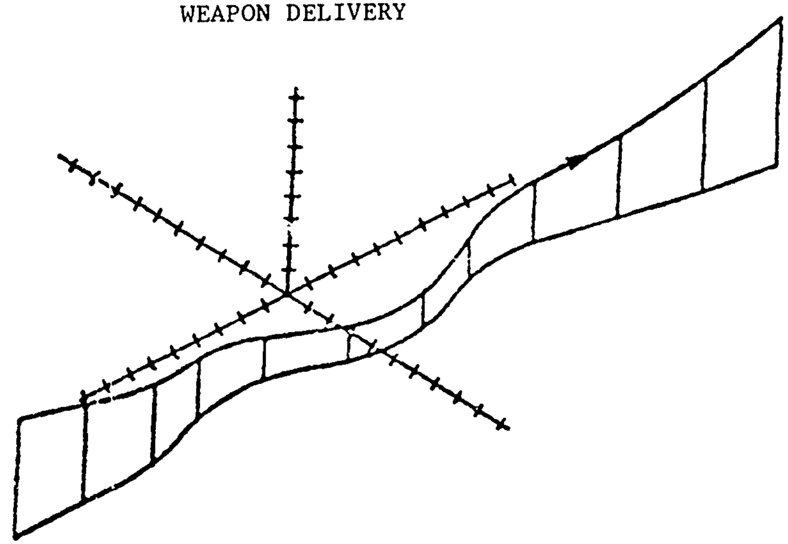
TRAJECTORY 1

5 X 5 FLYBY



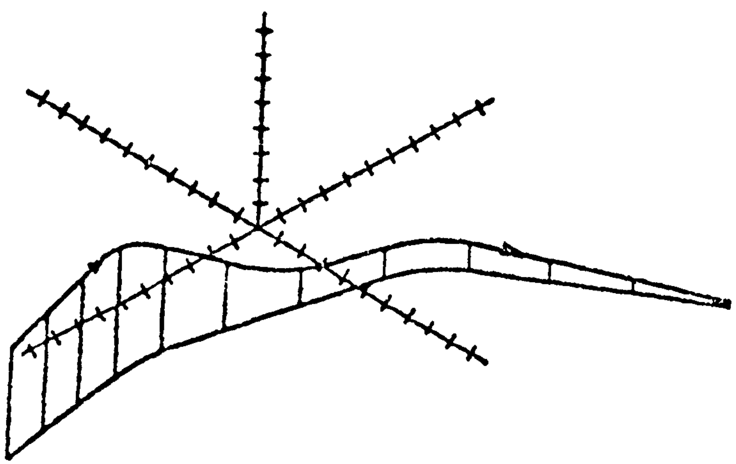
TRAJECTORY 2

WEAPON DELIVERY



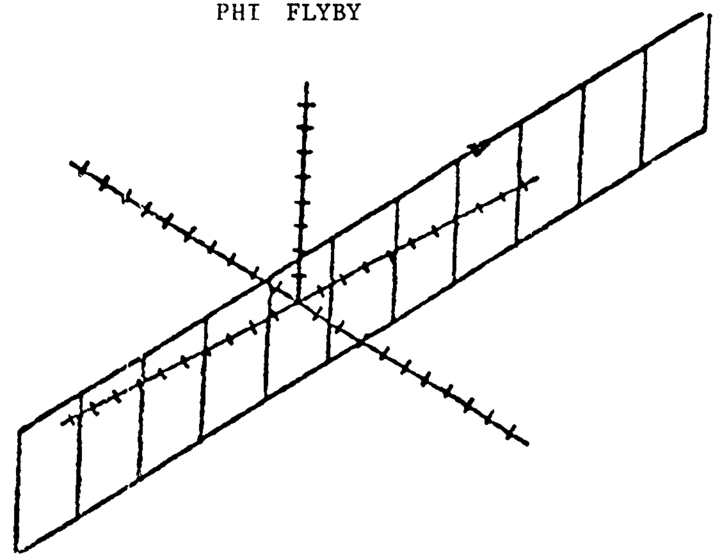
TRAJECTORY 3

FAIRPASS



TRAJECTORY 4

PHI FLYBY



Equation (1) is equivalent to $\dot{\theta}_g(t) = u(t)$. Furthermore, Figure 1 defines the tracking error $e_T(t)$ to be $\theta_T(t) - \theta_g(t)$. The time derivative of $e_T(t)$ is:

$$\begin{aligned}\dot{e}_T(t) &= \dot{\theta}_T(t) - \dot{\theta}_g(t)\\ &= \dot{\theta}_T(t) - u(t)\end{aligned}$$

Now let us introduce state variables

$$\begin{aligned}x_1(t) &\stackrel{\Delta}{=} \dot{\theta}_T(t)\\ x_2(t) &\stackrel{\Delta}{=} e_T(t) = \theta_T(t) - \theta_g(t)\end{aligned}$$

The time derivatives of these state variables are

$$\begin{aligned}\dot{x}_1(t) &= \ddot{\theta}_T(t)\\ \dot{x}_2(t) &= x_1(t) - u(t)\end{aligned}$$

Let $\underline{x}(t)$ be $[x_1(t), x_2(t)]^T$. The state space equation of the gunsight dynamics and target motion can be expressed as

$$\dot{\underline{x}}(t) = A\underline{x}(t) + Bu(t) + F\ddot{\theta}_T(t) \tag{2}$$

where A, B, and F are matrices defined as follows:

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad F = \begin{bmatrix} 1 \\ 0 \end{bmatrix} .$$

The scalars u and $\ddot{\theta}_T$ are the control output of an AAA gunner and the target acceleration respectively.

The tracker's observation of the tracking error is represented in the measurement equation:

$$y(t) = C\underline{x}(t) \tag{3}$$

where C is a row vector [0 1].

B. Human Operator's Internal Model

To develop the observer model for the human operator's tracking response, it is necessary to obtain his internal model of the controlled plant and target motion, Figure 3. This internal model describes the understanding or knowledge the gunner (i.e. human operator) has about the real system

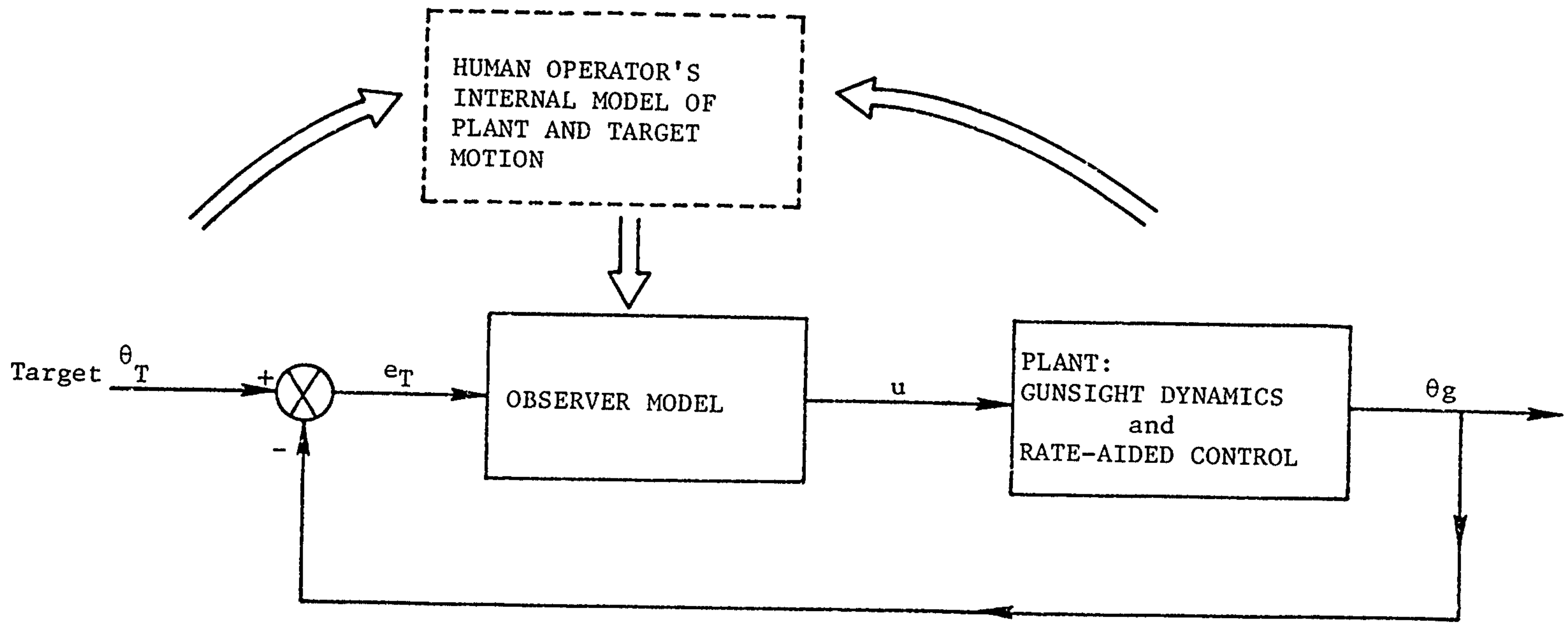


FIGURE 3: BLOCK DIAGRAM OF DESIGN PHILOSOPHY

(including gunsight dynamics, display, control crank, and target motion). For simplicity, it is assumed that the human operator's internal model of gunsight dynamics is identical to the mathematical model of gunsight dynamics.

As mentioned, the tracker has information about target trajectory θ_T, and maybe target velocity $\dot{\theta}_T$, but not precise information of the target acceleration $\ddot{\theta}_T$. Hence, the last term, $\ddot{\theta}_T$, in Eq. (1) will not be included in the human operator's internal model of the system. This is the target uncertainty problem whose effect will be included in a remnant element, considered in the next section.

In addition, the observation noise which might be associated with measured output y will not be considered explicitly. Its random effect will also be included in the remnant element for simplicity. Therefore, the human operator's internal model of the system can be described as follows:

$$\begin{aligned} \underline{\dot{x}}_1(t) &= A\underline{x}_1(t) + Bu(t) \\ y(t) &= C\underline{x}(t) \end{aligned} \tag{4}$$

The state variable $\underline{x}_1(t)$ of the internal model also has dimension two.

C. Observer Model

An observer is itself a dynamic system whose function is to reconstruct the state variable of a given system (in this case, the human operator's internal model). An identity observer is an observer which has the same dynamic order as the observed system. In this report, an identity observer is designed to estimate the states of the gunsight and the target motion. The structure of an identity observer used in our observer model design is:

$$\begin{aligned} \underline{\dot{z}}(t) &= A\underline{z}(t) + Bu_c(t) + K\,[y(t) - C\underline{z}(t)] \\ &= (A-KC)\underline{z}(t) + Bu_c(t) + Ky(t) \end{aligned} \tag{5}$$

$\underline{z}(t)$ is the state variable of the observer with dimension two. This vector represents the estimated value of the state variable $\underline{x}(t)$ and is used to fulfill the state variable feedback controller. Matrices A, B, and C are as previously

described. K is the observer gain vector. The dynamic response of the observer is determined by the matrix A-KC. K is designed such that the eigenvalues of A-KC have negative real parts (i.e. A-KC is a stable matrix). Then the state of the observer will converge to the state of the observed system. According to observer theory, there always exists a gain K such that A-KC is stable if the system is observable. The definition of an observable system can be formed in [10]. It can be shown that the system described by Eq.(4) is observable. $u_c(t)$ in Eq.(5) is the feedback controller and is defined as follows:

$$u_c(t) = -\Gamma \underline{\mathbf{z}}(t) \tag{6}$$

where Γ is the controller gain, a row vector with two elements. In Eqs. (5) and (6), the observer gain K and the controller gain Γ will be determined by a parameter identification program.

The relation between the control output u(t) of the human operator and $u_c(t)$ is:

$$u(t) = u_c(t) + v(t)$$

where v(t) is a random process called remnant element which represents all the gunner-induced noises, (e.g., the effect of target uncertainty, the observation noise, the neuromotor noise, etc.) and modeling errors. The idea to lump all the random effects into the remnant element is to simplify the structure of the model. The statistical properties of the remnant v(t) are:

$$E[v(t)] = 0 \qquad \text{for all } t,$$

and (7)

$$E[v(t)\, v^T(\tau)] = V(t)\delta(t-\tau) \text{ for all } t \text{ and } \tau.$$

where $E[\cdot]$ denotes the expectation value of $\cdot$, δ is the Dirac delta function and V(t) is a function of time to be described in section III. A block diagram of the structure of the observer model is shown in Figure 4. Let $\underline{e}(t)$ be the

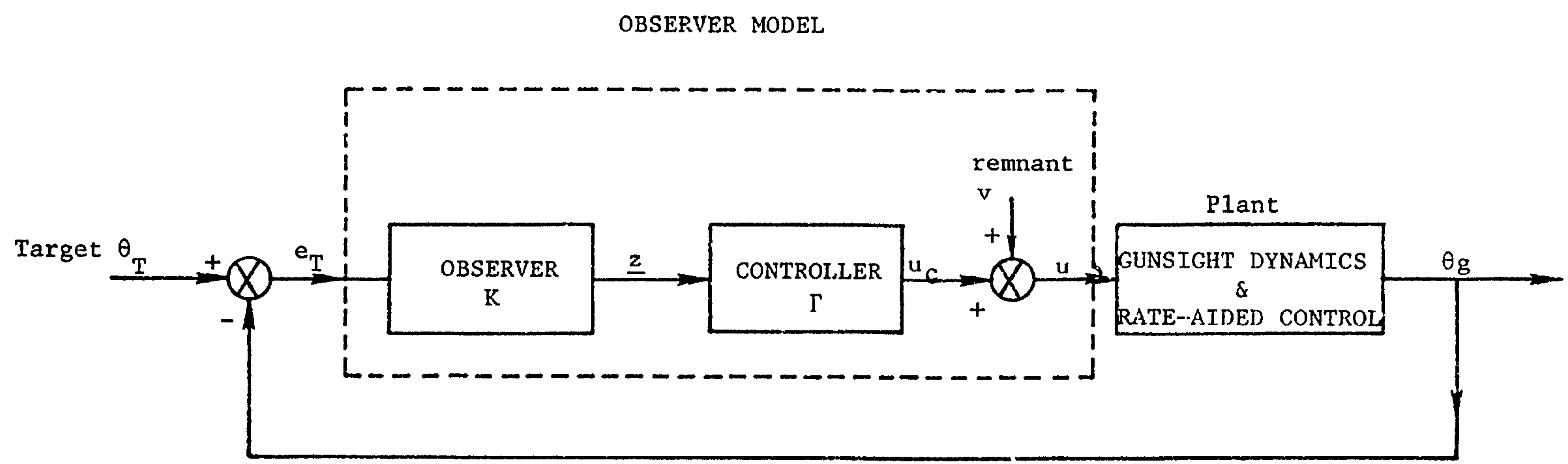


FIGURE 4: BLOCK DIAGRAM OF OBSERVER MODEL STRUCTURE

estimation error, i.e., $\underline{e}(t) = \underline{x}(t) - \underline{z}(t)$, then

$$\dot{\underline{e}}(t) = \dot{\underline{x}}(t) - \dot{\underline{z}}(t)$$

and from Eqs. (2) and (5)

$$\dot{\underline{e}}(t) = (A-KC)\underline{e}(t) + Bv(t) + F\ddot{\theta}_T(t) \tag{8}$$

K will be selected to make A-KC a stable matrix. Hence, $\underline{e}(t)$ will decrease exponentially to zero with time. The mathematical model of the AAA closed loop system which is composed of Eqs. (2) and (5) can be written as:

$$\begin{aligned} \dot{\underline{x}}(t) &= (A-B\Gamma)\underline{x}(t) + B\Gamma\underline{e}(t) + Bv(t) + F\ddot{\theta}_T(t) \\ \dot{\underline{e}}(t) &= (A-KC)\underline{e}(t) + Bv(t) + F\ddot{\theta}_T(t) \end{aligned} \tag{9}$$

Let

$$\underline{z}_1(t) = \begin{bmatrix} \underline{x}(t) \\ \underline{e}(t) \end{bmatrix},$$

$$A_1 = \begin{bmatrix} A-B\Gamma & B\Gamma \\ 0 & A-KC \end{bmatrix}, \quad B_1 = \begin{bmatrix} B \\ B \end{bmatrix} \text{ and } F_1 = \begin{bmatrix} F \\ F \end{bmatrix}$$

Then

$$\dot{\underline{z}}_1(t) = A_1\underline{z}_1(t) + B_1v(t) + F_1\ddot{\theta}_T(t) \tag{10}$$

Note that the system matrix A_1 of the above overall system is a triangular matrix. Hence, the eigenvalues of this triangular matrix are the eigenvalues of matrices $A-B\Gamma$ and A-KC. Now, if we choose a proper control gain matrix Γ and observer gain matrix K to make $A-B\Gamma$ and A-KC stable matrices then the overall system is stable. Furthermore, it can be shown that the design of observer and the design of controller can be done separately. This simplifies the design procedures. The mean of Eq. (10) is:

$$\dot{\bar{\underline{z}}}_1(t) = A_1\bar{\underline{z}}_1(t) + F_1\ddot{\theta}_T(t) \tag{11}$$

where $\bar{\underline{z}}_1(t) = E[\underline{z}_1(t)]$.

The covariance of this equation is:

$$\dot{P}(t) = A_1P(t) + P(t)A_1^T + B_1V(t)B_1^T \tag{12}$$

where $P(t) = E[(\underline{z}_1(t) - \bar{\underline{z}}_1(t))(\underline{z}_1(t) - \bar{\underline{z}}_1(t))^T]$. The derivation of Eq. (12) can be found in Reference [11].

SECTION III

PARAMETER IDENTIFICATION METHOD

The structure of the gunner model has been designed. To implement this work, we need to determine the values of the model parameters. A systematic procedure has been developed (Figure 5). It involves an identification program based on the least squares curve-fitting method and the Gauss-Newton gradient algorithm as described in Appendix B. One requirement of this technique is to select a criterion function $J(\underline{a})$ of the unknown parameters $\underline{a}$ to evaluate the "goodness of fit" between the model predictions from Eqs. (11) and (12) and the empirical data obtained from the manned AAA simulation. Parameter identification occurs iteratively by converging to a set of values for which the curves are "reasonably" matched.

Empirical data of the tracking error, $e_T(t)$, was collected at the Aerospace Medical Research Laboratory, WPAFB, Ohio, from their manned simulator using simulated target trajectory θ_T as input. $\bar{e}_T$ and SD_T represent the sample ensemble mean and standard deviation of the tracking errors over sixteen simulation runs for a given team. In Eq. (11) the model prediction of the ensemble mean of the tracking error, $\bar{e}'_T$ is located in the second component of $\bar{\underline{z}}_1(t)$. It can also be shown that the square root of the second diagonal element of the covariance matrix P(t) in Eq. (12) is the predicted ensemble standard deviation of the tracking error. The parameter identification was done in two parts (curve-fitting of the ensemble mean and ensemble standard deviation of the tracking error). For both parts, angular information was input only from Trajectory 4 of Figure 2.

In the first part, $\bar{e}_T$ and $\bar{e}'_T$ are compared to determine the observer gain $K = [k_1, k_2]$ and the controller gain $\Gamma = [\gamma_1, \gamma_2]$. This comparison was actually computed in frequency domain. The power spectral density function (PSD) of the sample ensemble mean tracking error was calculated, $S_{\bar{e}_T}(\omega)$. And the PSD, $S_{\bar{e}'_T}(\omega)$, of $\bar{e}'_T$ can be computed from the state representation of $\bar{\underline{z}}_1(t)$ in Eq. (11). The

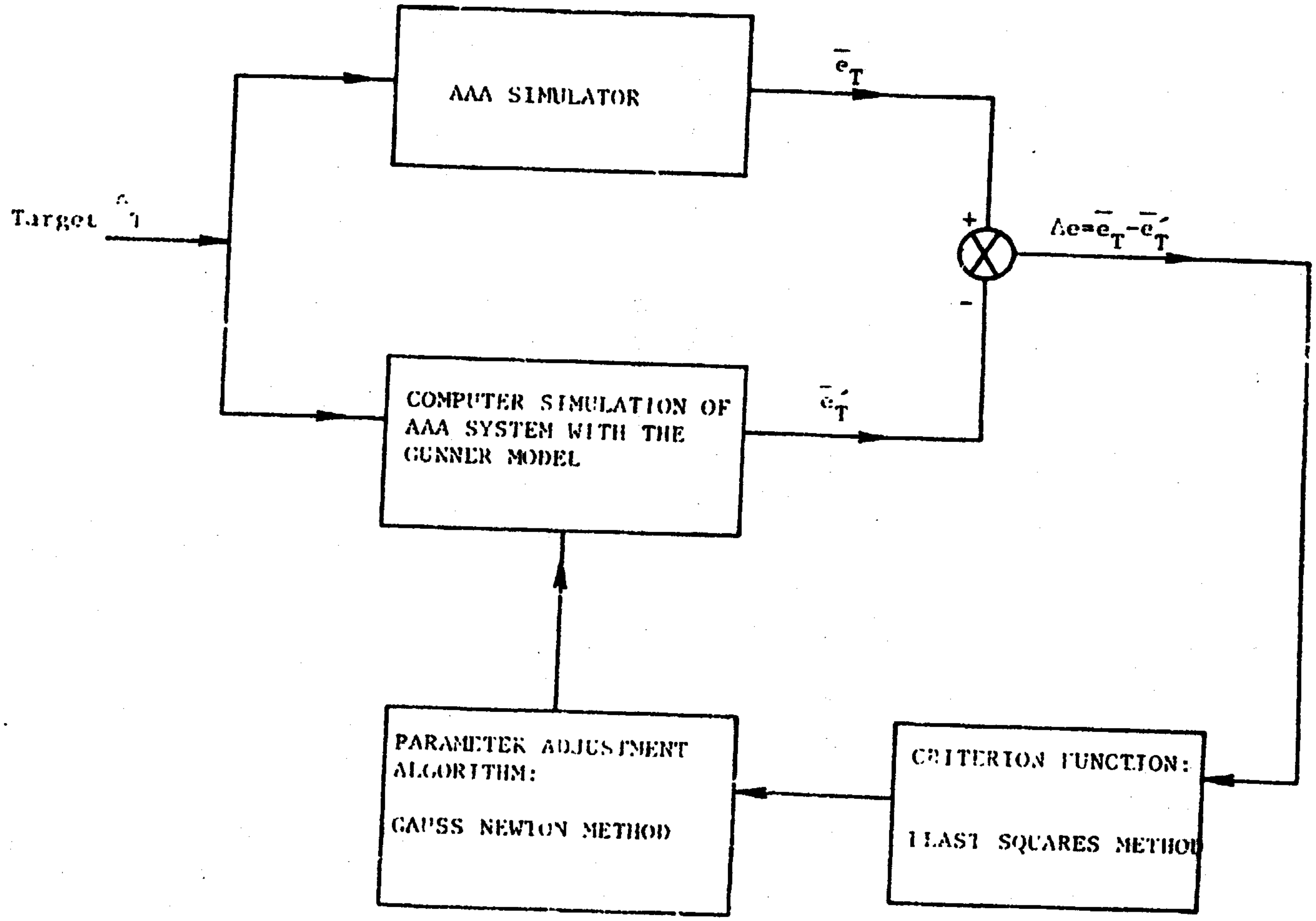


FIGURE 5: BLOCK DIAGRAM OF THE PARAMETER IDENTIFICATION PROCEDURE

solution to this differential equation is:

$$\bar{z}_1(t) = \int_{-\infty}^{t} \phi(t-\tau)F_1\ddot{\theta}_T(\tau)d\tau \tag{13}$$

where $\phi(t) = \mathrm{EXP}(A_1 t)$ is the transition matrix.

Then

$$\bar{e}_T'(t) = \int_{-\infty}^{t} [\phi_{21}(t-\tau) + \phi_{23}(t-\tau)]\ \ddot{\theta}_T(\tau)d\tau \tag{14}$$

where $\phi_{ij}(t)$ is the ij^{th} element of the matrix $\phi(t)$. The PSD of $\bar{e}_T'(t)$, denoted by $S_{\bar{e}_T'}(\omega)$ can be shown [10] to be

$$\begin{aligned} S_{\bar{e}_T'}(\omega) &= |G_{21}(j\omega)|^2\ S_{\ddot{\theta}_T}(\omega) + |G_{23}(j\omega)|^2\ S_{\ddot{\theta}_T}(\omega) \\ &= [\,|G_{21}(j\omega)|^2 + |G_{23}(j\omega)|^2\,]\ S_{\ddot{\theta}_T}(\omega) \end{aligned} \tag{15}$$

where $S_{\ddot{\theta}_T}(\omega)$ is the PSD of $\ddot{\theta}_T(t)$ and $G_{ij}(j\omega)$ is the ij^{th} element of the square matrix $G(j\omega)$ which is defined:

$$G(s) = L\,[\phi(t)] \tag{16}$$

i.e., $G(s)$ is the Laplace transform of the transition matrix $\phi(t)$ and s is the variable of the complex plane. It can be shown that $G(s) = (sI-A_1)^{-1}$ hence can be rewritten as:

$$G(s) = \frac{\mathrm{adj}\ (sI-A_1)}{\det\ (sI-A_1)} \tag{17}$$

where I is the identity matrix, det and adj denote the determinant and adjoint of a matrix respectively.

Furthermore, because of the structure of A_1, it can be shown that (see reference [8])

$$G(s) = \frac{\mathrm{adj}\ (sI-A_1)}{\det\,[sI-(A-B\Gamma)]\cdot\det\,[sI-(A-KC)]} \tag{18}$$

Hence

$$G_{21}(s) = \frac{1+\gamma_1}{s(s-\gamma_2)}$$

$$G_{23}(s) = \frac{\gamma_1 s + \gamma_1 k_2 + \gamma_2}{(s-\gamma_2)\ (s^2 + k_2 s + k_1)}$$

and

$$|G_{21}(j\omega)|^2 = \frac{(1+\gamma_1)^2}{\omega^2(\omega^2+\gamma_2^2)} \tag{19}$$

$$|G_{23}(j\omega)|^2 = \frac{(\gamma_1\omega)^2 + (\gamma_1 k_1 + \gamma_2)^2}{[\omega^3 - \omega(k_1 - \gamma_2 k_2)]^2 + [(k_2 - \gamma_2)\omega^2 + \gamma_2 k_1]^2} \tag{20}$$

But, when ω=0, $|G_{21}(j\omega)|^2$ is undefined unless $\gamma_1 = -1$. Under this condition, we now have

$$S_{\bar{e}_T'}(\omega) = \frac{\omega^2 + (\gamma_2 - k_2)^2}{[\omega^2 - \omega(k_1 - \gamma_2 k_2)]^2 + [(k_2 - \gamma_2)\omega^2 + \gamma_2 k_1]^2} \cdot S_{\ddot{\theta}_T}(\omega) \tag{21}$$

Since we normalized all PDS's, this becomes

$$S'_{\bar{e}_T'}(\omega) = \frac{(\gamma_2 k_1)^2}{(\gamma_2 - k_2)^2} \cdot S_{\bar{e}_T'}(\omega) \tag{22}$$

Let $\underline{a} = [\gamma_2, k_1, k_2]$ with $\gamma_1 = -1$, then compute the partial derivatives of $S'_{\bar{e}_T'}(\omega,\underline{a})$ with respect to $\underline{a}$.

$$\frac{\partial S'_{\bar{e}_T}(\omega, \underline{a})}{\partial \underline{a}}$$

All the information is now ready for implementation of the curve fitting program. The criterion function to be minimized is:

$$J(\underline{a}) = \int_0^{\omega_f} [S'_{\bar{e}_T}(\omega) - S'_{\bar{e}_T'}(\omega, \underline{a})]^2 d\omega \tag{23}$$

where ω_f is the frequency bandwidth of the target trajectory (input) and $S'_{\bar{e}_T}(\omega)$ is the normalized PSD of the sample ensemble mean of the tracking error. For the computer program we used the discretized form

$$J(\underline{a}) \cong \sum_{k=1}^{K} [S'_{\ddot{e}_T}(\omega_k) - S'_{\bar{e}_T'}(\omega_k, \underline{a})]^2 \Delta\omega \tag{24}$$

where $\Delta\omega$ is the chosen increment between samples in the frequency domain and $\omega_k = k\cdot\Delta\omega$, $\omega_f = K\Delta\omega$. The iterative equation to be solved by the program is

$$\underline{a}_{i+1} = \underline{a}_i - \rho\left[\sum_{k=1}^{K} \left(\frac{\partial S_{e_T}^{'}(\omega_k, \underline{a}_i)}{\partial \underline{a}} \right)^T \cdot \left(\frac{\partial S_{e_T}^{'}(\omega_k, \underline{a}_i)}{\partial \underline{a}} \right) \right]^{-1}$$

$$\cdot \left\{ \sum_{k=1}^{K} \left(\frac{\partial S_{e_T}^{'}(\omega_k, \underline{a}_i)}{\partial \underline{a}} \right)^T \left[S_{e_T}^{'}(\omega_k, \underline{a}_i) - S_{e_T}^{'}(\omega_k) \right] \right\} \qquad (25)$$

where ρ is a step size factor defined in Appendix B. The values of model parameters obtained through the identification program are listed in Table 1 below:

	AZIMUTH	ELEVATION
γ_1	-1	-1
γ_2	-1.77	-1.877
k_1	25	25
k_2	2.6	3.76

Table 1

Once these gains are determined, the system matrix is known. The second part of curve fitting involves comparing the model prediction and the sample ensemble standard deviation SD_T of the tracking error. This part of the program was computed in time domain.

To begin, we need the solution to Eq. (12). (See reference [11].)

$$P(t) = \phi(t, t_0)\, P(t_0)\, \phi^T(t, t_0) + \int_{t_0}^{t} \phi(t,\tau) B_1 V(\tau) B_1^T \phi^T(t,\tau)\, d\tau \qquad (26)$$

Select $P(t_0)$ to be a diagonal matrix with diagonal elements d_{ii}, $i = 1, 2, \cdots, n$. Thus, the second diagonal element $P_{22}(t)$ of the covariance matrix $P(t)$ is

$$P_{22}(t) = \sum_{i=1}^{n} d_{ii}\phi_{2i}^{2}(t-t_0) + \int_{t_0}^{t} \sum_{j=1}^{n} \sum_{i=1}^{n} \phi_{2i}(t-\tau)B_1 V(\tau)B_1^T \phi_{2j}(t-\tau)d\tau$$

$$= \sum_{i=1}^{n} d_{ii}\phi_{2i}^{2}(t-t_0) + \int_{t_0}^{t} [\phi_{22}(t-\tau) + \phi_{24}(t-\tau)]^2 V(\tau)d\tau \tag{27}$$

Since K and Γ are known and since $G_{22}(s)$ and $G_{24}(s)$ can be obtained from Eq.(19), we need only take the inverse Laplace transform of these quantities to obtain:

$$\phi_{22}(t) = e^{\gamma_2 t}$$

$$\phi_{24}(t) = -c_1 e^{-\gamma_2 t} + c_1 e^{-c_2 t}\cos c_3 t + c_4 e^{-c_2 t}\sin c_3 t \tag{28}$$

with c_i defined in Table 2 below

	AZIMUTH	ELEVATION
c_1	1.2	1.33
c_2	1.3	1.88
c_3	.26	.4
c_4	4.83	4.633

Table 2

V(t) is the covariance matrix of the random remnant. It has been found that the main source of tracking error is due to the gunner's uncertainty about the target trajectory dynamics especially the target velocity $\dot{\theta}_T$ and the target acceleration $\ddot{\theta}_T$. Furthermore, study of the curve-fitting between the empirical data and the model prediction of the ensemble standard deviation of the tracking error has indicated that the gunner's target motion uncertainty is the dominant part of the random remnant term v(t). Therefore, it is proposed that the covariance

function V(t) of the remnant term be a function of the target dynamics as follows:

$$V(t) = \alpha_1 + \alpha_2 \hat{\dot{\theta}}_T^2(t) + \alpha_3 \hat{\ddot{\theta}}_T^2(t) \tag{29}$$

where α_1, α_2 and α_3 are three nonnegative constants to be identified and $\hat{\dot{\theta}}_T$ and $\hat{\ddot{\theta}}_T$ are estimated target angle rate and acceleration, respectively. The reason that only estimated values $\hat{\dot{\theta}}_T$ and $\hat{\ddot{\theta}}_T$ were used to represent target dynamics is that the gunner doesn't have precise information about $\dot{\theta}_T$ and $\ddot{\theta}_T$ (i.e., target uncertainty problem). These estimated quantities can be obtained as follows:

$$\hat{\dot{\theta}}_T = (\bar{\underline{z}}_1)_1 - (\bar{\underline{z}}_1)_3 \tag{30}$$

where $\bar{\underline{z}}_1$ is the state of the closed loop AAA system with the observer model, and $(\bar{\underline{z}}_1)_1$ and $(\bar{\underline{z}}_1)_3$ are the first and third components of the vecor $\bar{\underline{z}}_1$. Then by approximation:

$$\hat{\ddot{\theta}}_T(t_k) \cong \frac{\hat{\dot{\theta}}_T(t_k) - \hat{\dot{\theta}}_T(t_{k-1})}{\Delta t} \tag{31}$$

where Δt is the sampling interval, $t_k = k \cdot \Delta t$. Let

$\underline{b} = [\alpha_1 \ \alpha_2 \ \alpha_3]$, then in addition to $P_{22}(t, \underline{b})$, we need

$$\frac{\partial P_{22}(t,\underline{b})}{\partial \underline{b}}$$

Finally, the criterion function:

$$J^1(\underline{b}) = \int_{t_0}^{t_f} [\, SD_T^2(t) - P_{22}(t, \underline{b}) \,]^2 dt \tag{32}$$

For the computer program, a discretized form of the criterion function is used.

$$J^1(\underline{b}) \cong \sum_{k=1}^{K} [\, SD_T^2(t_k) - P_{22}(t_k, \underline{b}) \,]^2 \cdot \Delta t \tag{33}$$

The iterative equation to be solved by the program is:

$$\underline{b}_{i+1} = \underline{b}_i - \rho \left[\sum_{k=1}^{K} \left(\frac{\partial P_{22}(t_k,\underline{b}_i)}{\partial \underline{b}}\right)^T \cdot \left(\frac{\partial P_{22}(t_k,\underline{b}_i)}{\partial \underline{b}}\right) \right]^{-1} \cdot \left[\sum_{k=1}^{K} \left(\frac{\partial P_{22}(t_k,\underline{b}_i)}{\partial \underline{b}}\right)^T \cdot \left(P_{22}(t_k,\underline{b}_i) - SD_T^2(t_k)\right) \right]$$

The values of model parameters obtained through the identification program are listed in Table 3.

	AZIMUTH	ELEVATION
α_1	.001	.0005
α_2	.025	.05
α_3	.008	.0025

Table 3

A block diagram and computer program listings for each of the two parameter identification parts (i.e., ensemble mean and ensemble standard deviation) can be found in Appendix C.

SECTION IV

COMPUTER SIMULATION RESULTS

A. Discretization of Observer Model Equations

To more efficiently use computer time and core, the observer model equations were first discretized and then programmed.

So, given the continuous state variable equation (10), discretization yields:

$$\underline{z}_{n+1} = e^{A_1 \Delta t}\underline{z}_n + \int_0^{\Delta t} e^{A_1(\Delta t - s)} F_1 \ddot{\theta}_T(t_n+s)ds + \int_0^{\Delta t} e^{A_1(\Delta t-s)} B_1 v(t_n+s)ds \quad (34)$$

$$\cong e^{A_1 \Delta t}\underline{z}_n + \int_0^{\Delta t} e^{A_1(\Delta t - s)} ds\, F_1 \ddot{\theta}_T(t_n) + \int_0^{\Delta t} e^{A_1(\Delta t-s)} ds\, B_1 v(t_n)$$

$$= e^{A_1 \Delta t}\underline{z}_n + \int_0^{\Delta t} e^{A_1 \sigma}\, d\sigma\, F_1 \ddot{\theta}_T(t_n) + \int_0^{\Delta t} e^{A_1 \sigma}\, d\sigma\, B_1 v(t_n)$$

Hence, we have the following difference equation:

$$\underline{z}_{n+1} = \Phi\, \underline{z}_n + \Gamma_1 \ddot{\theta}_{T,n} + \Gamma_2 v_n \quad (35)$$

where

$$\Phi = e^{A_1 \Delta t}, \Gamma_1 = \int_0^{\Delta t} e^{A_1\sigma} d\sigma \cdot F_1,\ \Gamma_2 = \int_0^{\Delta t} e^{A_1 \sigma} d\sigma \cdot B_1,$$

Δt is the sampling period, $\ddot{\theta}_{T,n} = \ddot{\theta}_T(t_n)$, and v_n is a random sequence with the following properties:

$$E\,[v_n] = 0$$

$$E\,[\,(v_n - E[v_n])(v_n - E[v_n])^T] = V_n = \frac{V(t)}{\Delta t} \quad (36)$$

where $V(t)$ is defined in Eq. (7).

Taking expectation value of both sides of Eq. (35). We get

$$\overline{\underline{z}}_{n+1} = \Phi\, \overline{\underline{z}}_n + \Gamma_1 \ddot{\theta}_{T,n} \quad (37)$$

where $\overline{\underline{z}}_{n+1} = E[\underline{z}_{n+1}]$.

The covariance of $\underline{z}_{n+1}$ is defined as:

$$X_{n+1} = E[(\underline{z}_{n+1} - \overline{\underline{z}}_{n+1})(\underline{z}_{n+1} - \overline{\underline{z}}_{n+1})^T].$$

It can be shown [11] that X_{n+1} is governed by the following equation

$$X_{n+1} = \Phi X_n \Phi^T + \Gamma_2 V_n \Gamma_2^T \tag{38}$$

The predicted ensemble mean tracking error can be obtained from the second element of $\bar{z}_{n+1}$ of Eq. (37), and the predicted covariance of the tracking error can be found in the second diagonal element of the covariance matrix X_{n+1} of Eq. (38).

B. Simulation Results

The numerical values of the parameters of this gunner model were determined in section III with respect to the gunsight dynamics system (Eq. (1)) and a deterministic target trajectory. Angular information from Trajectory 4 was the input to the curve-fitting program. Since the parameters have been selected, all the necessary matrices in Section IV A. are defined. So the AAA gunner model is now ready to be used for computer sumulation. A computer program, OMS, for simulating an AAA system with this model representing the gunner response was implemented. A block diagram and program listing of this simulation can be found in Appendix D. The input to this program is the trajectory of the target motion. Initially, only Trajectory 4 was used. The outputs are the model predictions of the ensemble mean and standard deviation of the tracking errors. The results for Trajectory 4 are plotted in Figures 6a through 6d. Each graph contains empirical data, observer model predictions and P001 formula. (P001 formula refers to a simple formula to predict tracking error used in the P001 attrition model program [12].)

Azimuth mean tracking error and standard deviation are shown in Figures 6a and 6b respectively. Similarly Figures 6c and 6d show results for elevation tracking errors. It is obvious that the predictions by the P001 ormula did not match empirical data well. However, matching between empirical data and observer model predictions was very good. All these results indicated that this AAA gunner model is able to represent the trend of the gunner response in the tracking task. It was also noted that the sharp peaks in the empirical data curves were not matched by the model predictions. This may be due to

the simplified gunsight dynamics used in designing this model or an insufficient number of runs used in generating the sample ensemble mean data. Further research concerning these problems is necessary. Next, this gunner model, with the same set of parameter values, was used in OMS to predict the tracking errors for three other target trajectories (1,2,3, in Figure 2). Figures 7, 8, 9 picture these results in the same format as listed above. All the simulation results show that this AAA gunner model with the same parameter values gives model predictions in good agreement with empirical data. Therefore, the observer model is a predictive model in the sense that it can be used to predict tracking errors of an AAA system for various target trajectories with similar frequency bandwidths. In addition, the observer model is also an adaptive model since its parameters depend on the gunsight dynamics and the target trajectory.

A comparison of the model prediction accuracy between the observer model and the optimal control model has been done for these 4 target trajectories. All the results showed that both models give accurate predictions of the tracking errors. Figures 10a through 10d show the ensemble mean and standard deviation of the tracking error as predicted by the optimal control model for both azimuth and elevation tracking task. Upon comparing Figures 6 and 10, it is obvious that the AAA gunner model (i.e., observer model) developed in this paper can predict the tracking errors as accurately as those by the optimal control model. The computer simulation time of the closed loop AAA system using the observer model is only 6.5 seconds, while 37 seconds of simulation time are needed to execute the optimal control model. Therefore, a reduction of 85% computer simulation time can be obtained by using the observer model instead of the optimal control model. So this AAA gunner model based on the observer theory is very useful in the analysis of the performance of the AAA gun system.

SECTION V

CONCLUSION

The Luenberger observer theory has been applied to design an AAA gunner model. The key design requirement was to make the model structure simple so that it needs much less computer simulation time than the other models. It was also important to predict the tracking error accurately. Both specifications have been met. In addition, this report has presented a parameter identification program which can easily determine the numerical values of the parameters of this model. Then the model is ready for the computer simulation of the AAA gun system. The Aerospace Medical Research Laboratory, WPAFB, has applied this model to the study of a foreign AAA gun system. This model is now being applied to study several other foreign AAA gun systems and SAMs. The identity observer used in the observer model still possesses a certain degree of redundancy. The redundancy stems from the fact that the identity observer approximately constructs an estimate of the entire state but part of the state, as given by the system outputs, are already available for direct measurement. This redundancy can be eliminated by the use of a reduced-order observer to replace the identity observer. Further research is continuing to develop an observer model which uses fewer states. This will further simplify the structure of the current observer model and shorten the computer time. Now, to determine the parameter values for the model, it is necessary to have empirical tracking data from a given weapon system available. So another worthwhile extension of this work is to develop parameter adjusting rules for this model such that the numerical values of the parameters can be obtained without the use of the empirical data. This project is now under study.

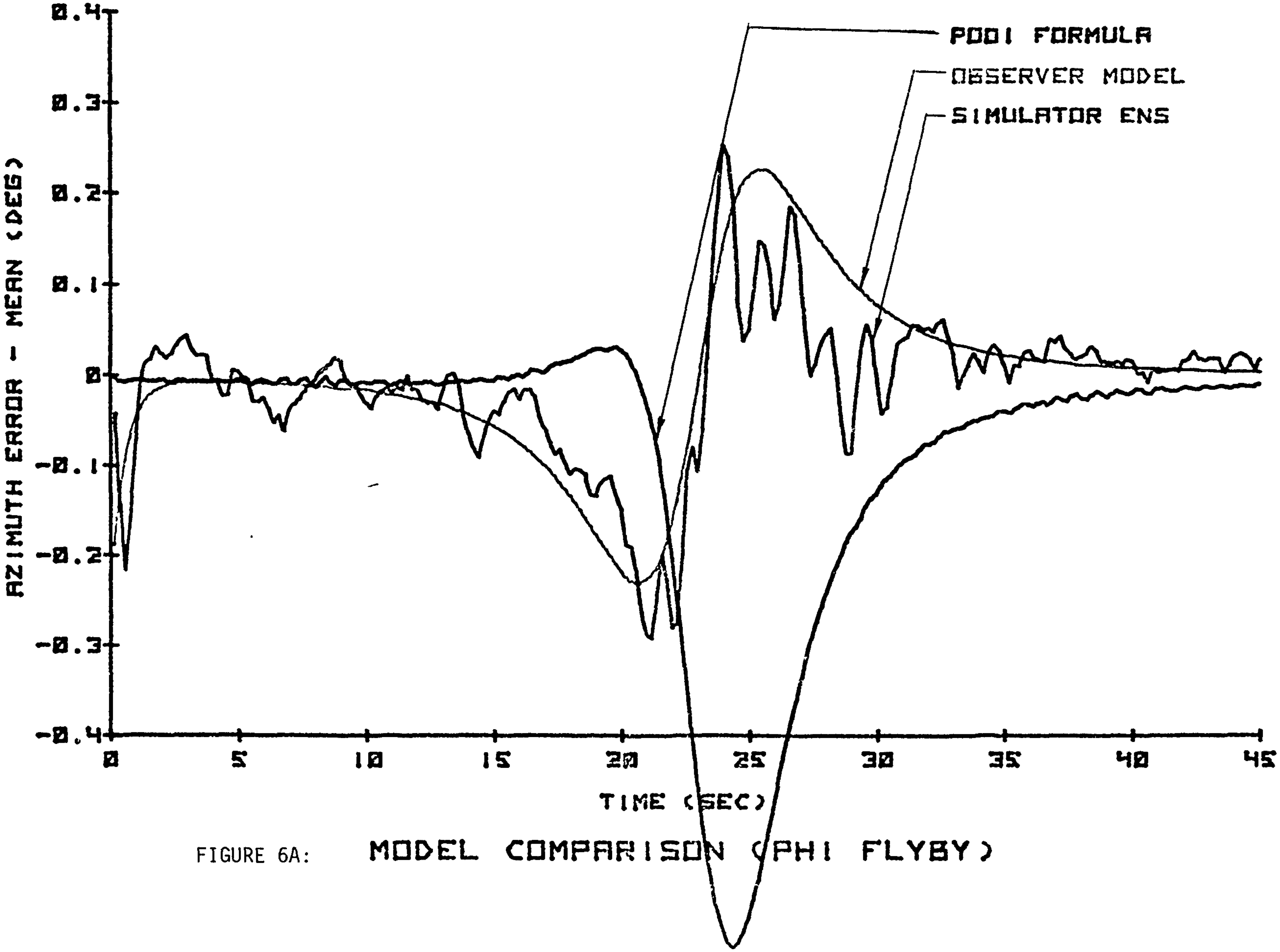


FIGURE 6A: MODEL COMPARISON (PHI FLYBY)

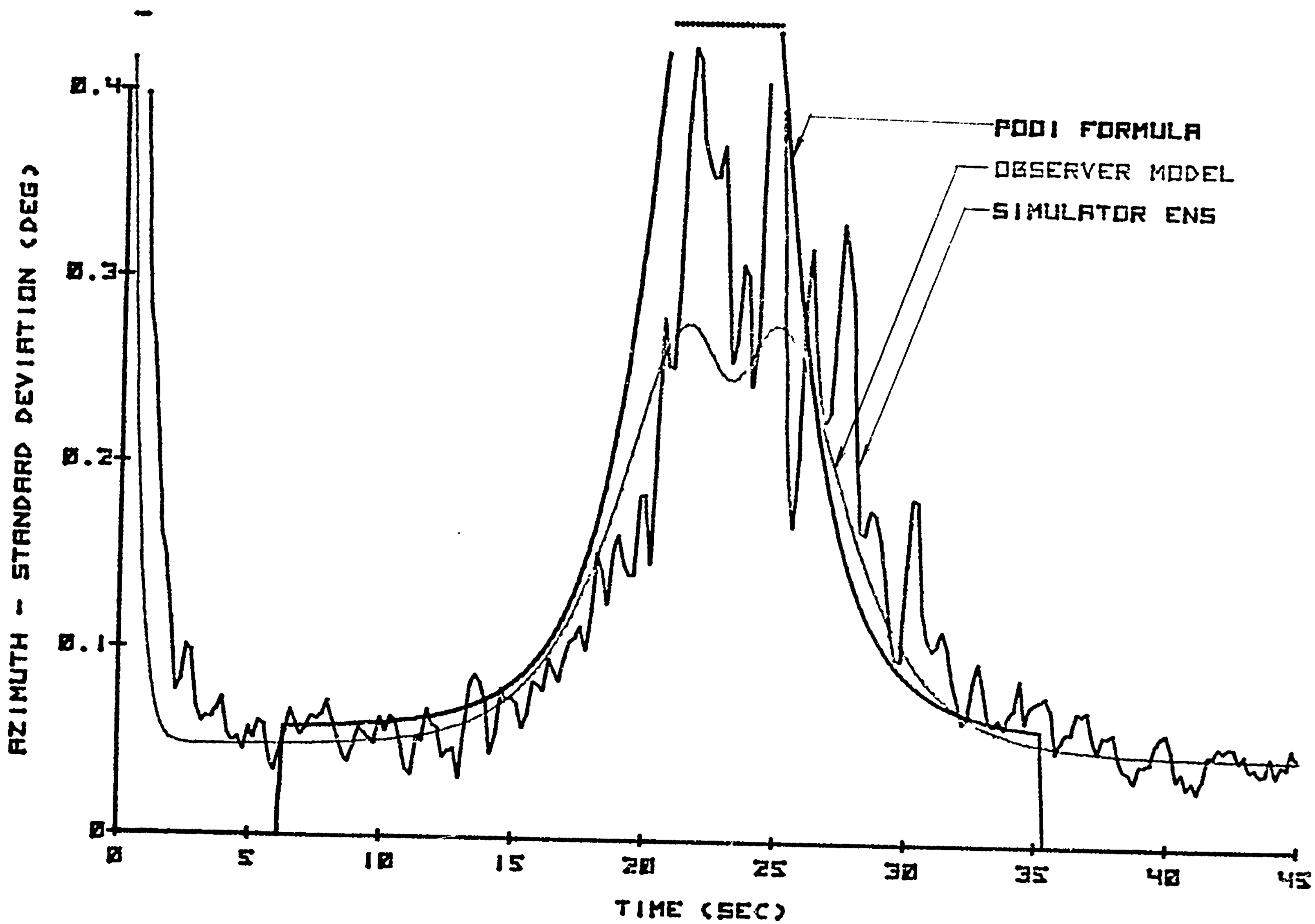


FIGURE 6B: MODEL COMPARISON (PHI FLYBY)

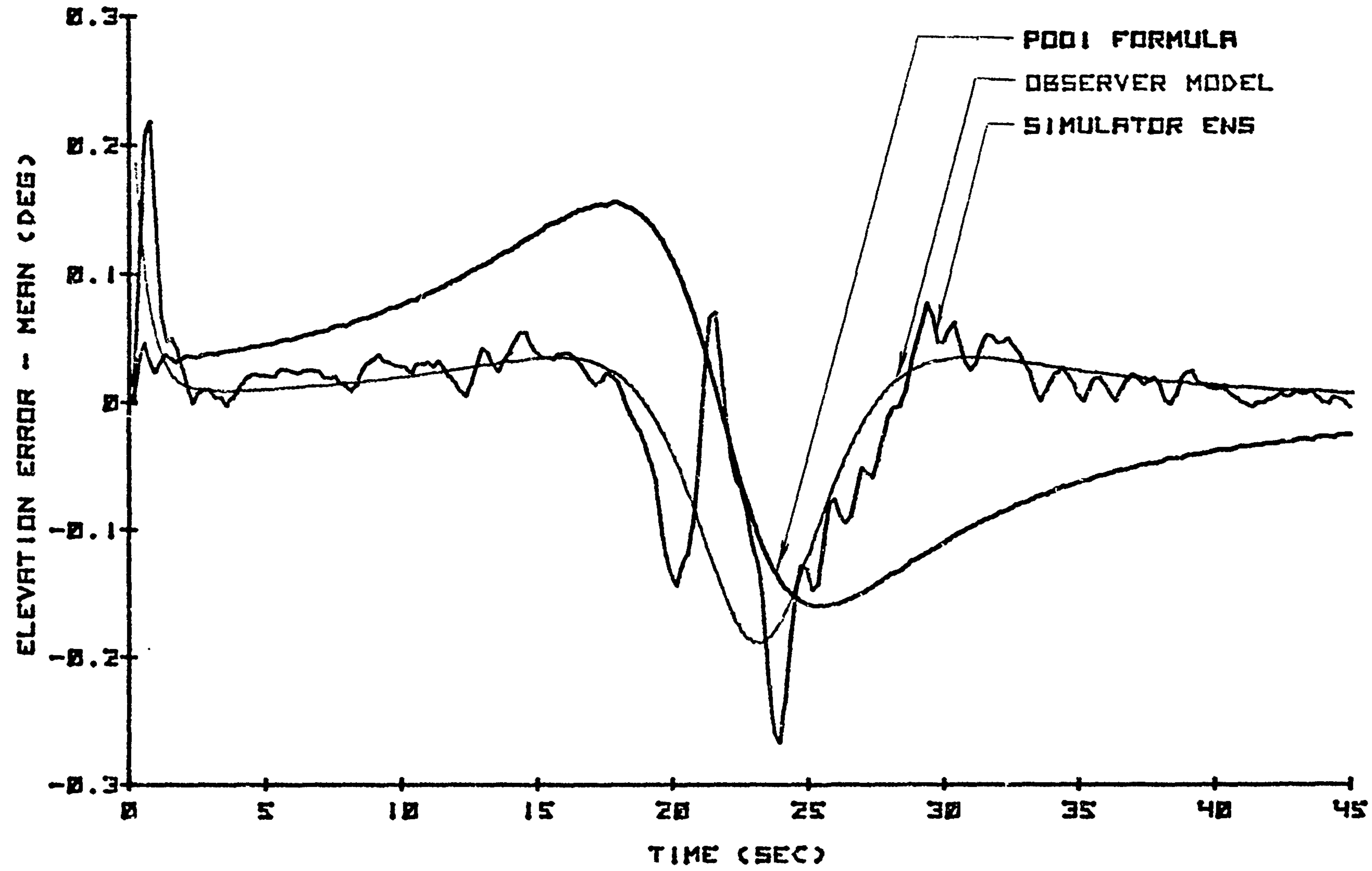


FIGURE 6C: MODEL COMPARISON (PHI FLYBY)

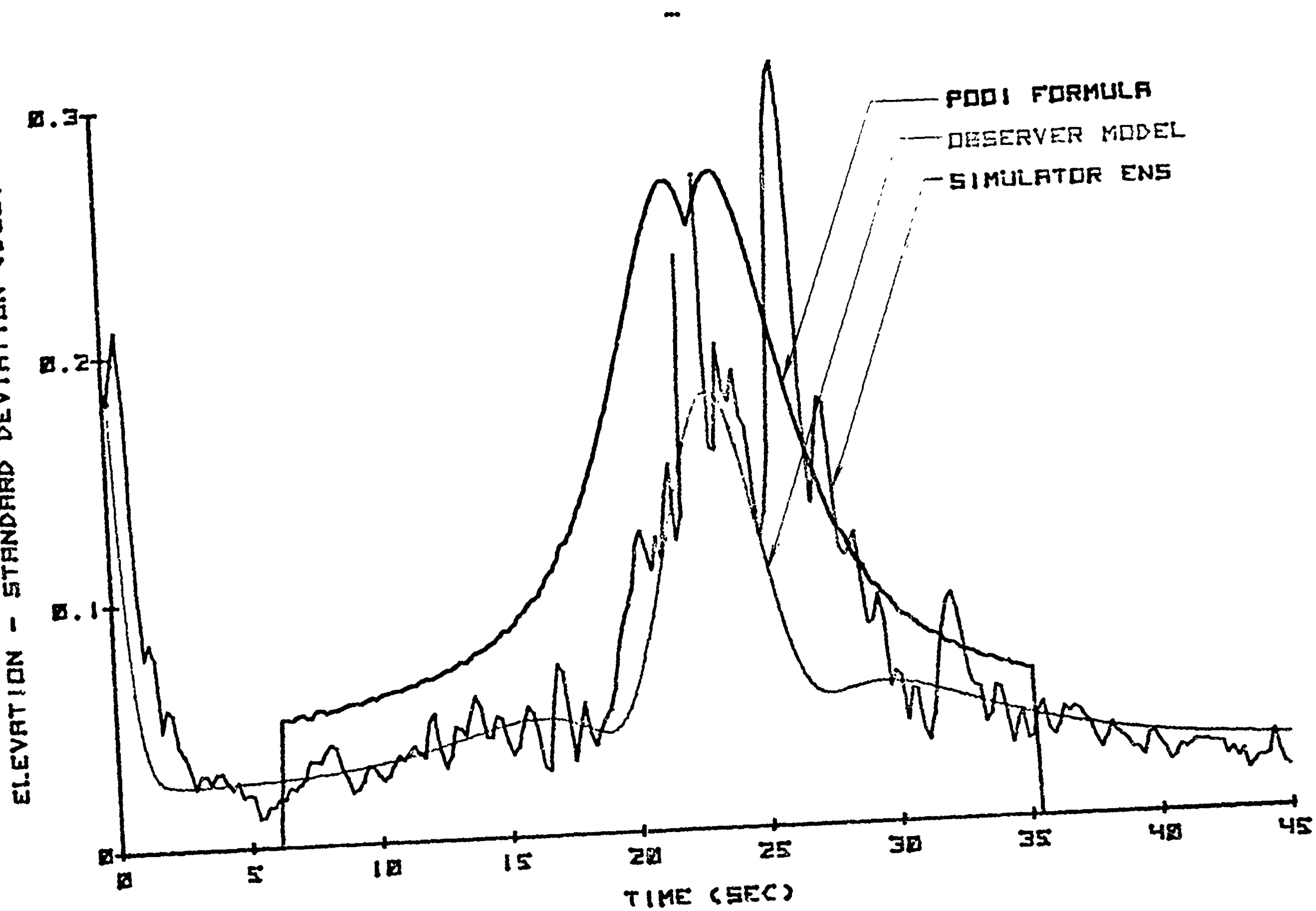


FIGURE 6D: MODEL COMPARISON (PHI FLYBY)

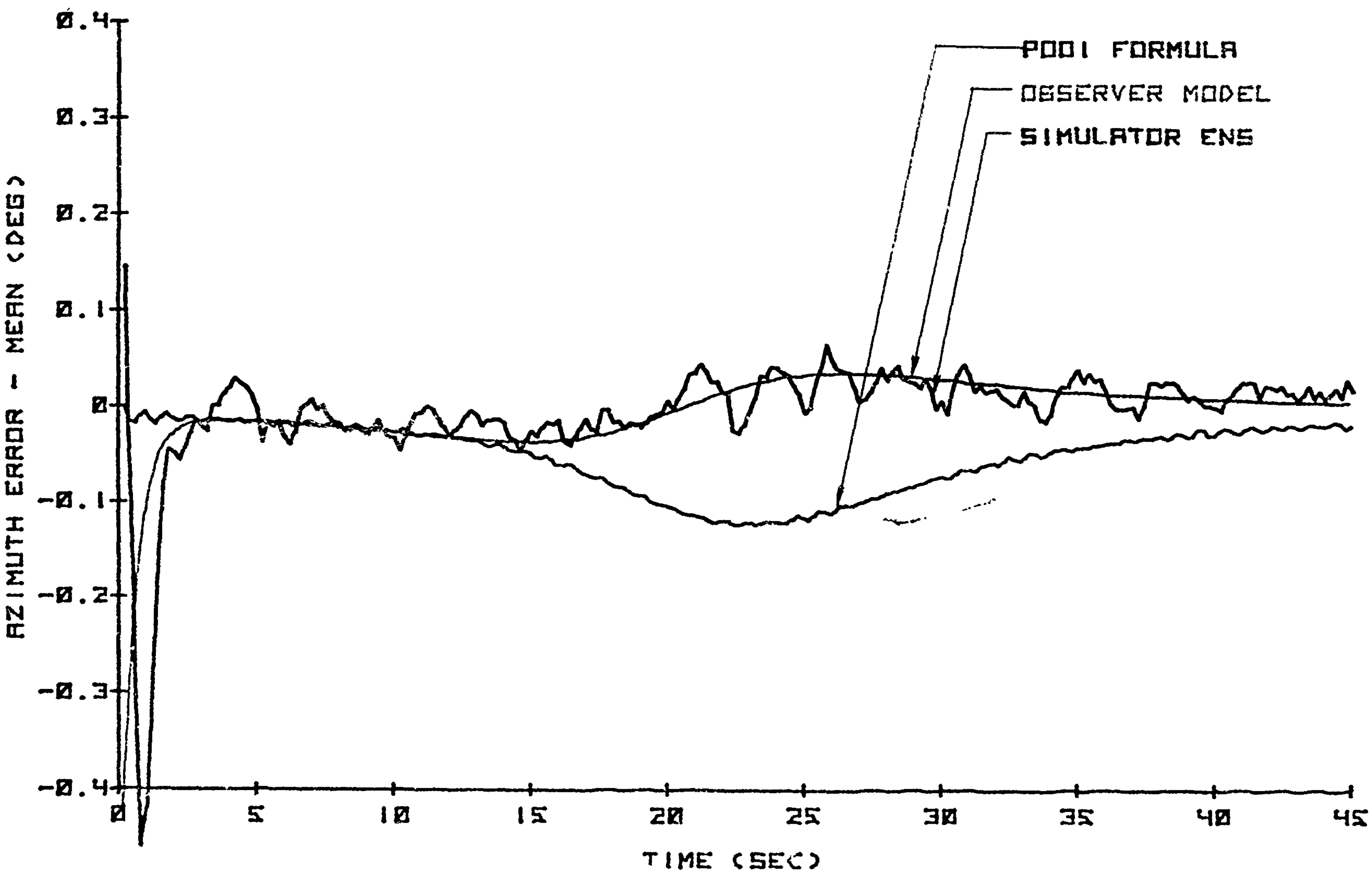


FIGURE 7A: MODEL COMPARISON (5 X 5 FLYBY)

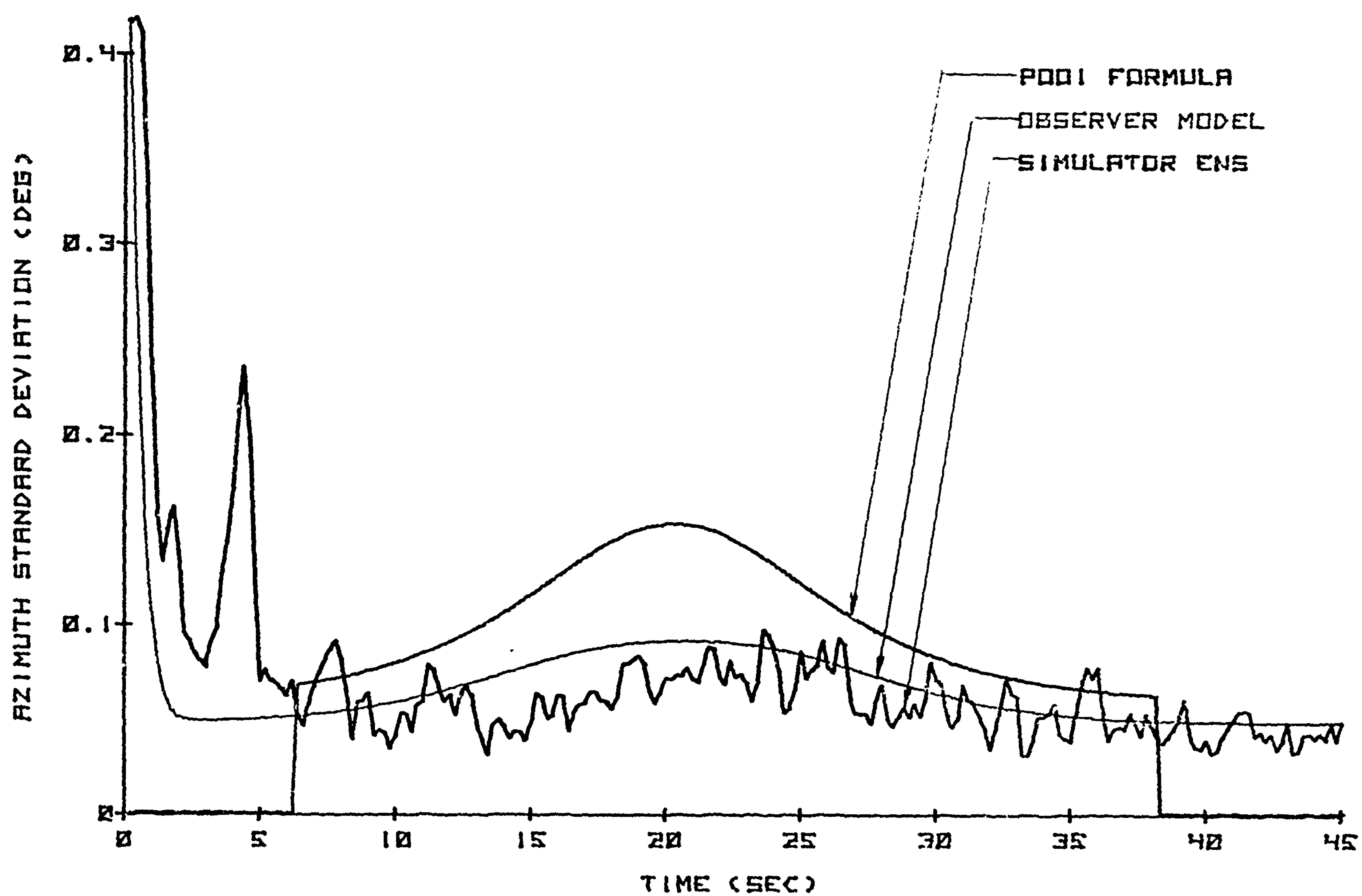


FIGURE 7B: MODEL COMPARISON (5 X 5) FLYBY

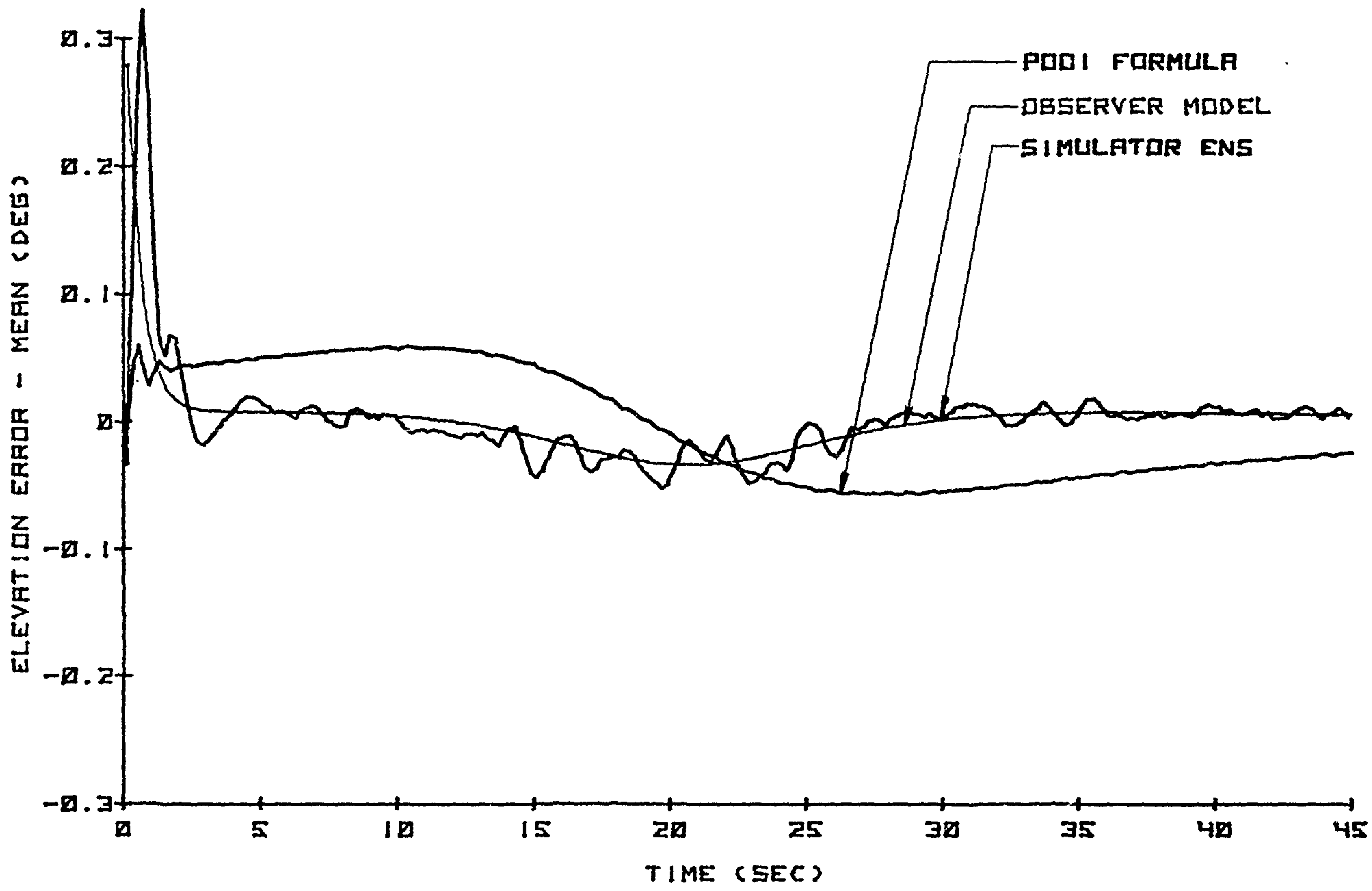


FIGURE 7C: MODEL COMPARISON (SXS FLYBY)

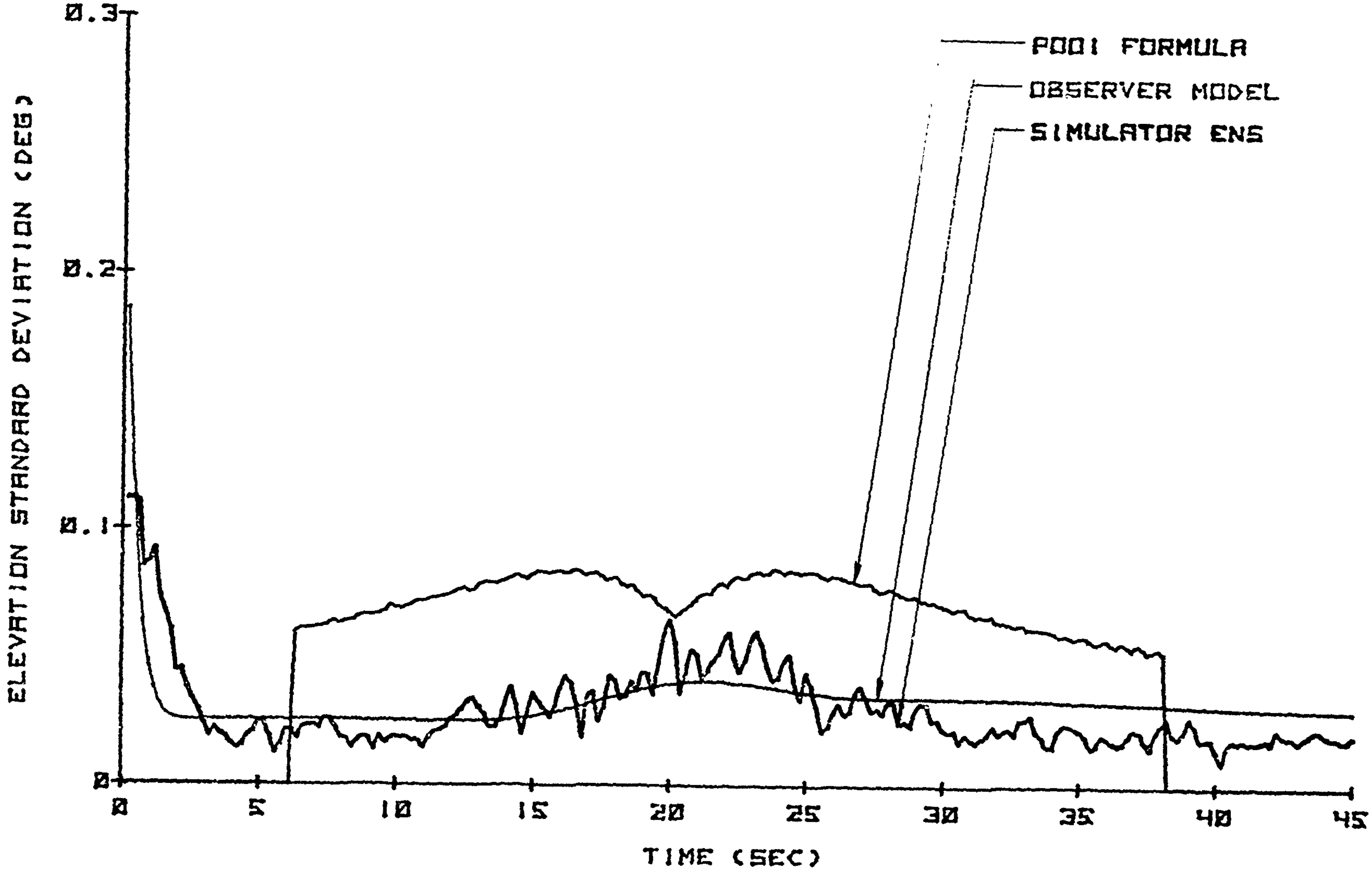


FIGURE 7D: MODEL COMPARISON (5 X 5) FLYBY

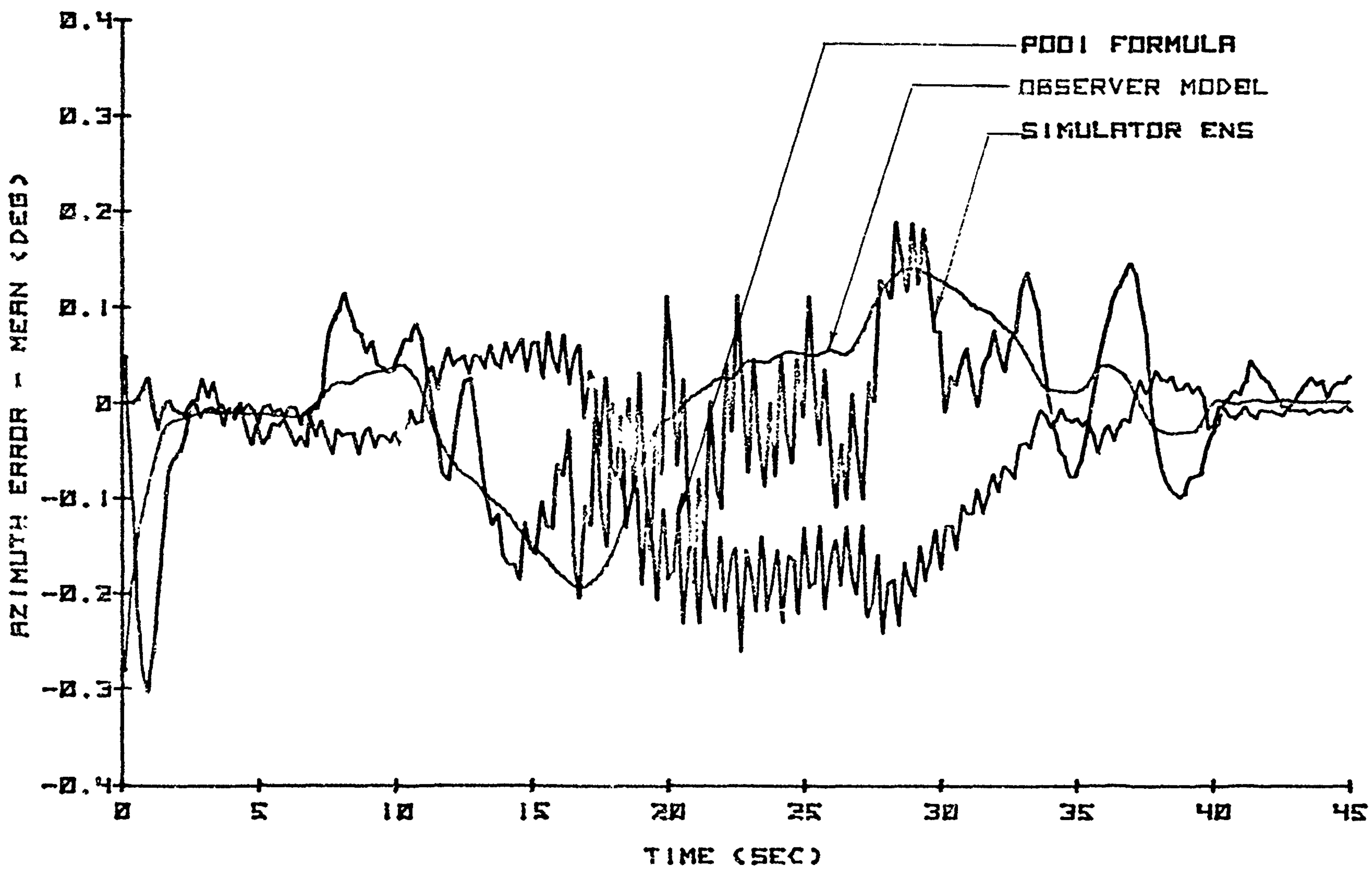


FIGURE 8A: **MODEL COMPARISON (WEAPON DELIVERY)**

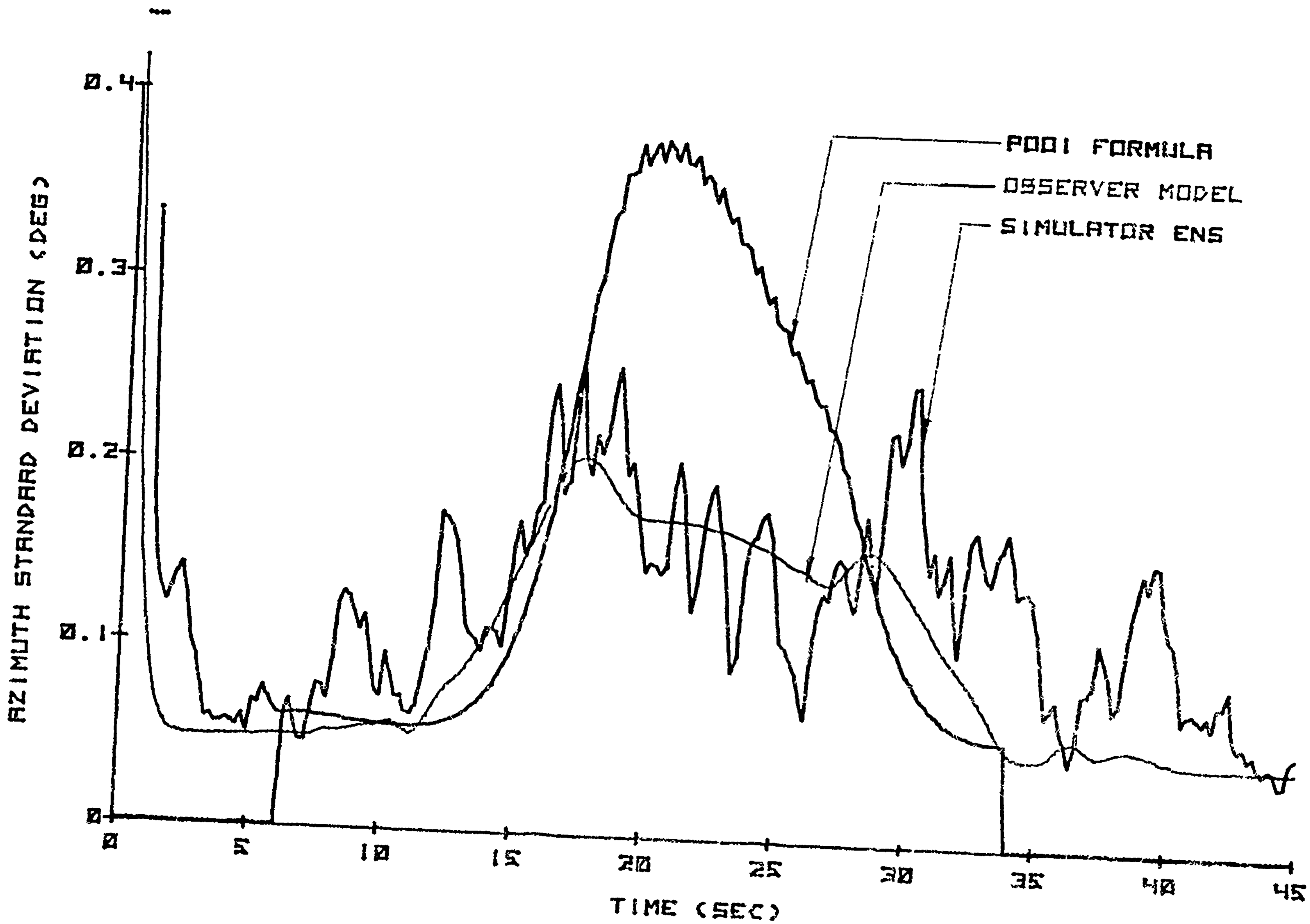


FIGURE 86: MODEL COMPARISON (WEAPON DELIVERY)

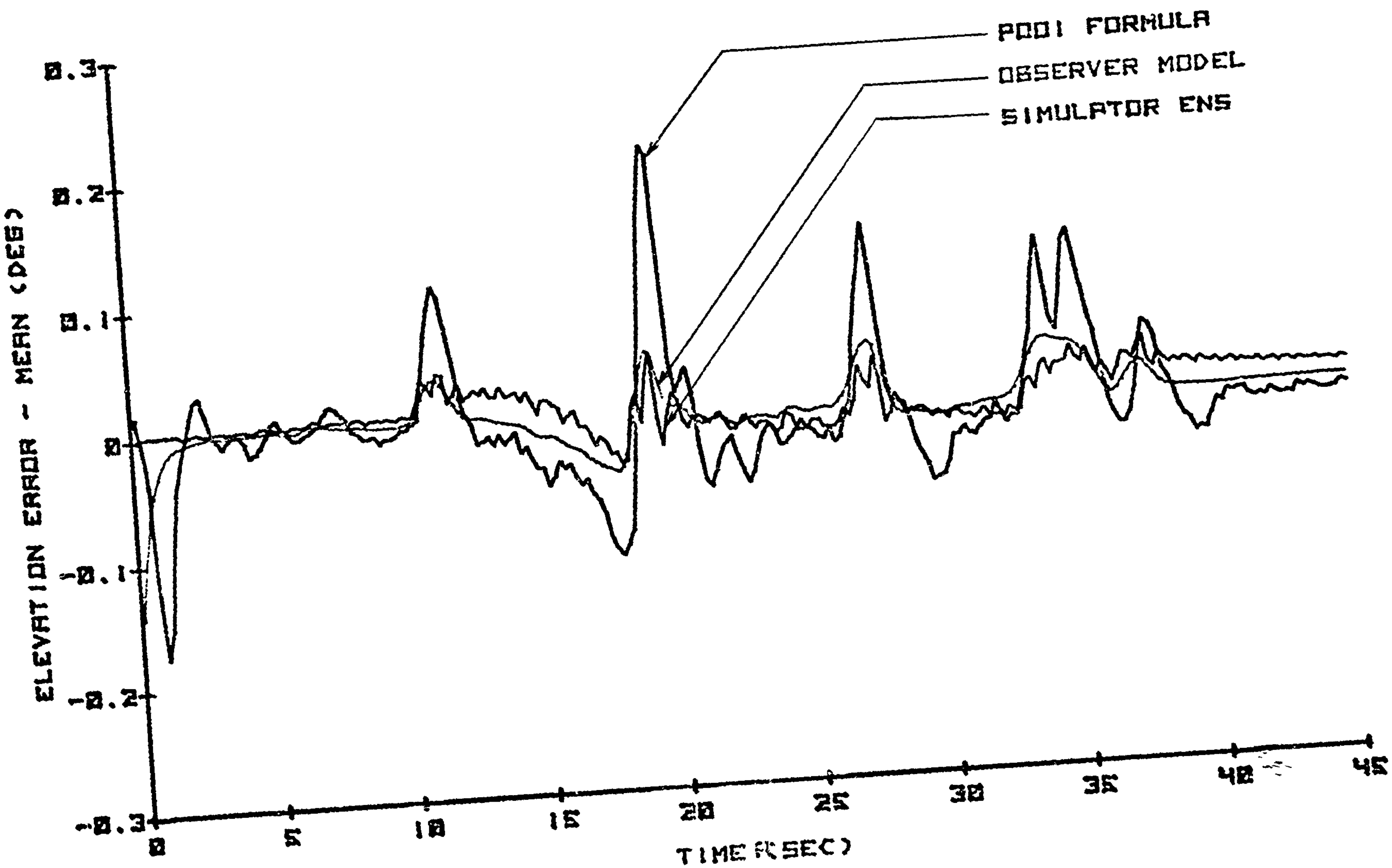


FIGURE 8C: MODEL COMPARISON (WEAPON DELIVERY)

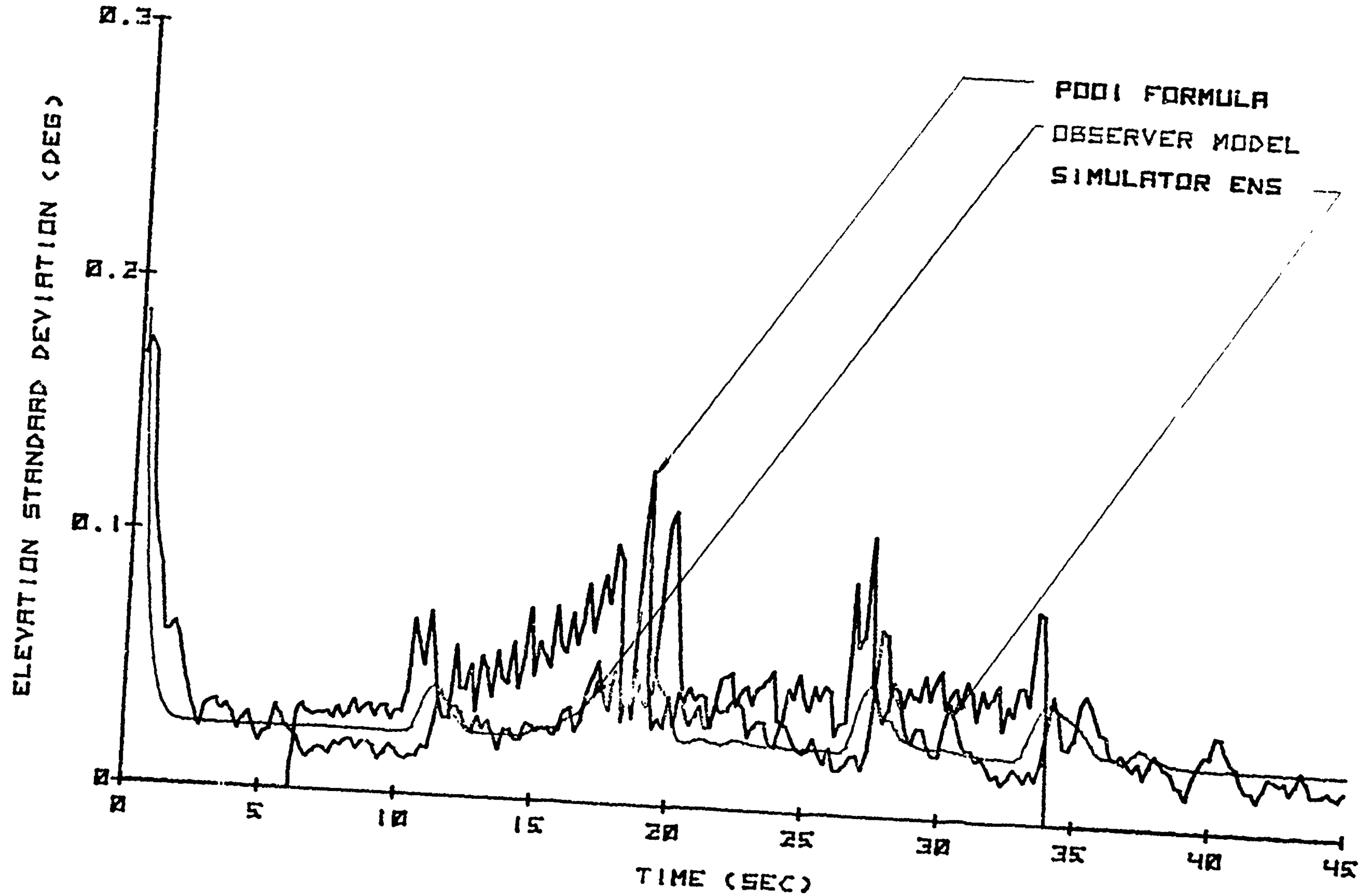


FIGURE 8D: MODEL COMPARISON (WEAPON DELIVERY)

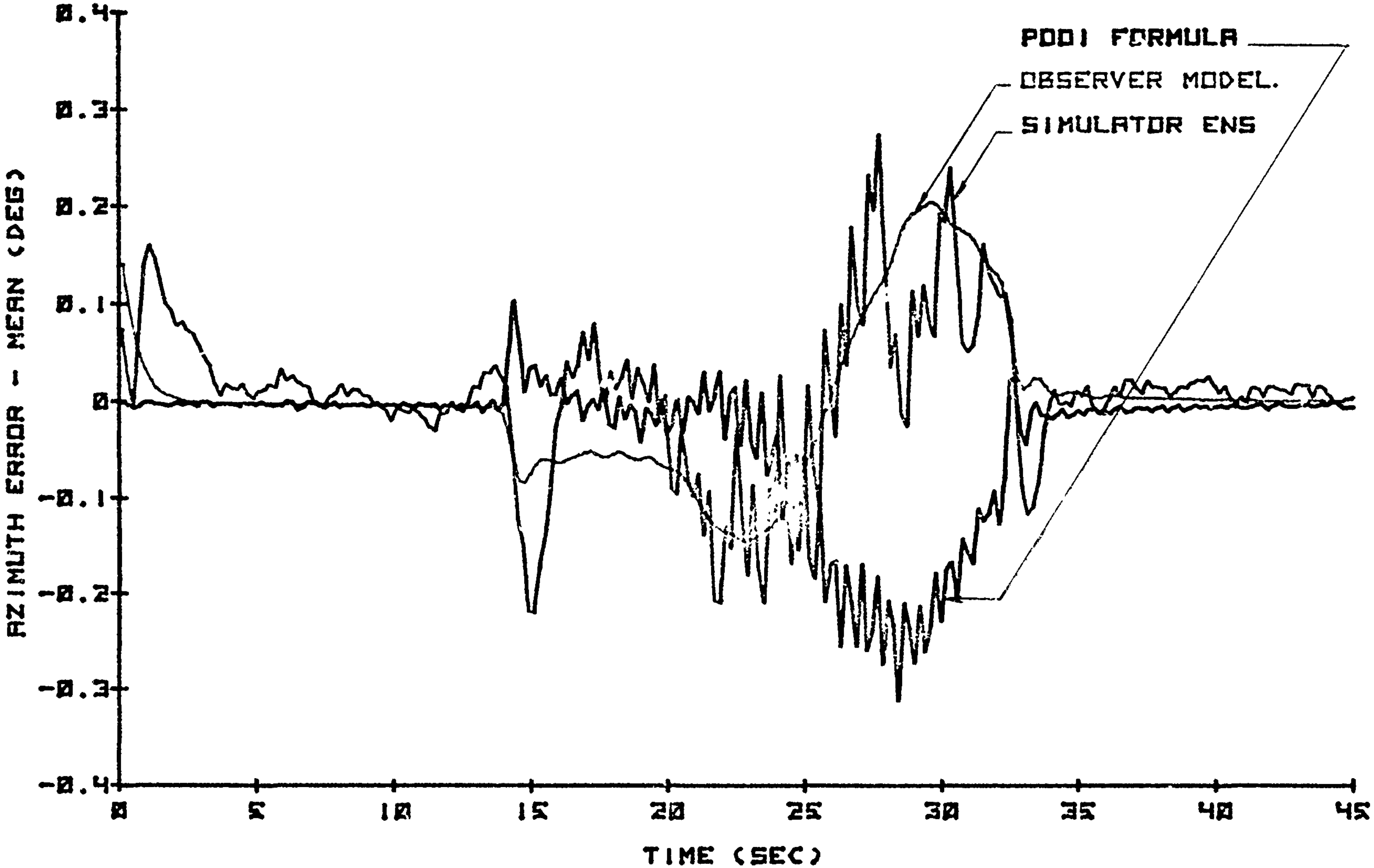


FIGURE 9A: MODEL COMPARISON (FAIR PASS)

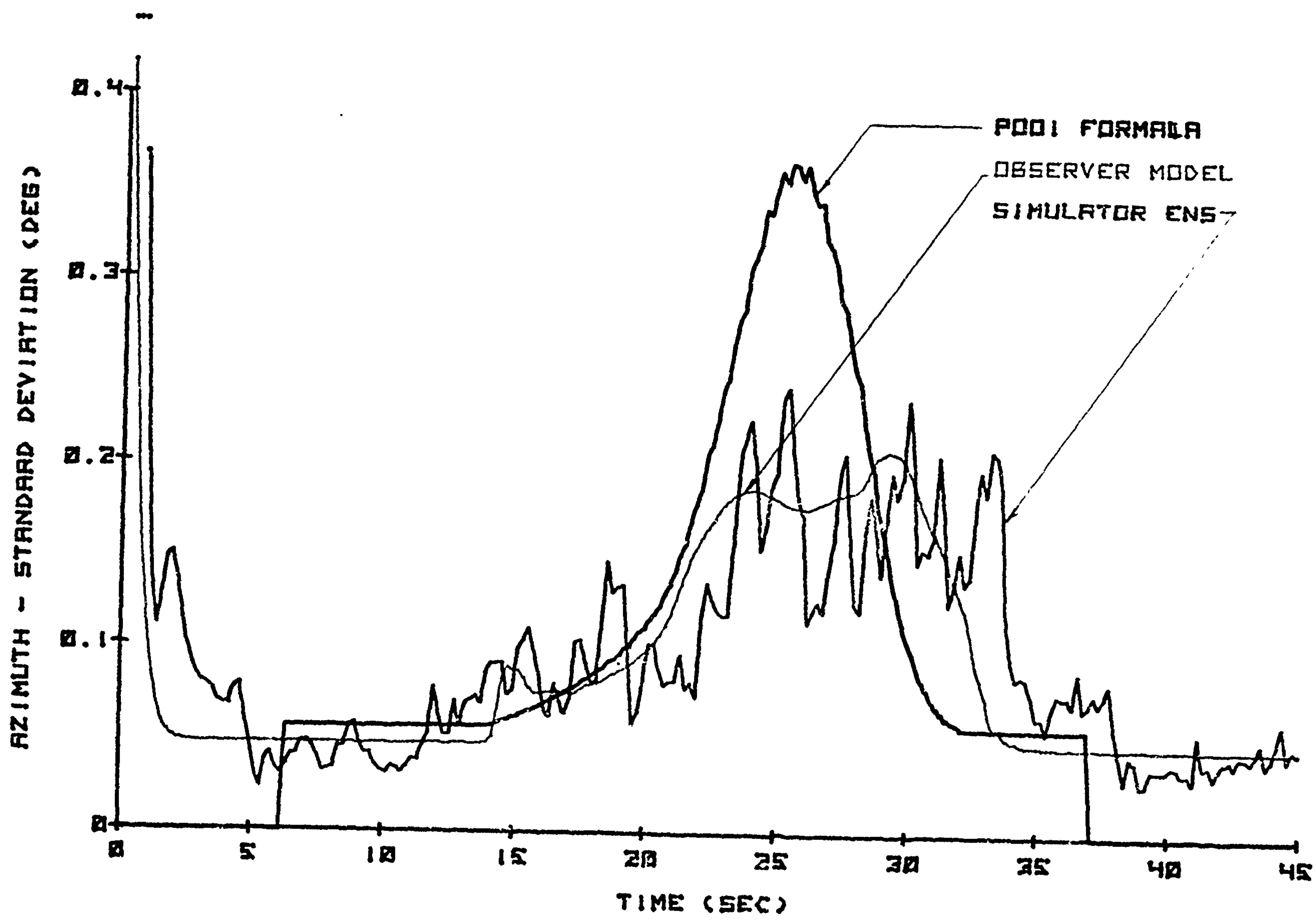


FIGURE 9B: MODEL COMPARISON (FAIR PASS)

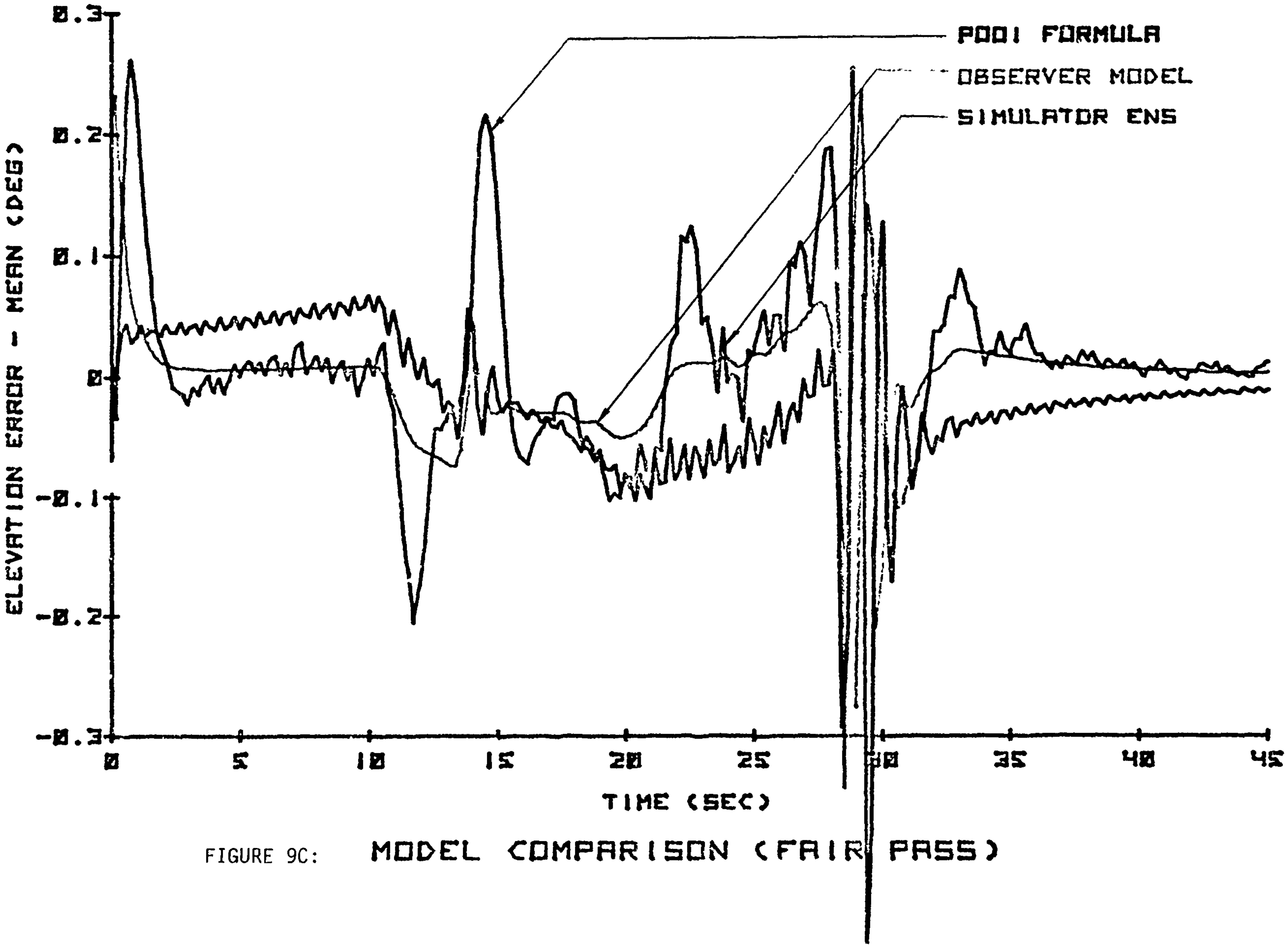


FIGURE 9C: MODEL COMPARISON (FAIR PASS)

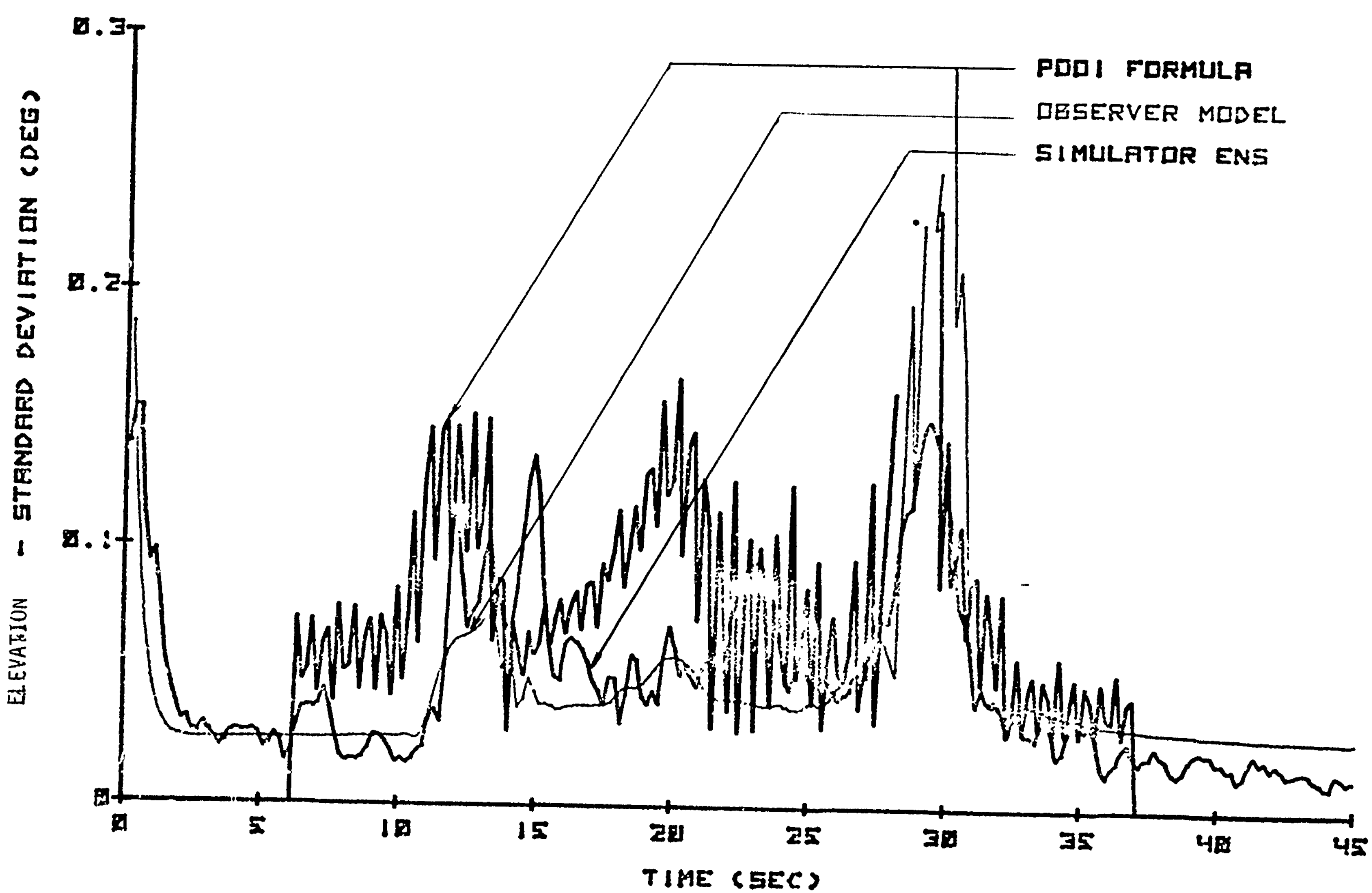


FIGURE 9D: MODEL COMPARISON (FAIR PASS)

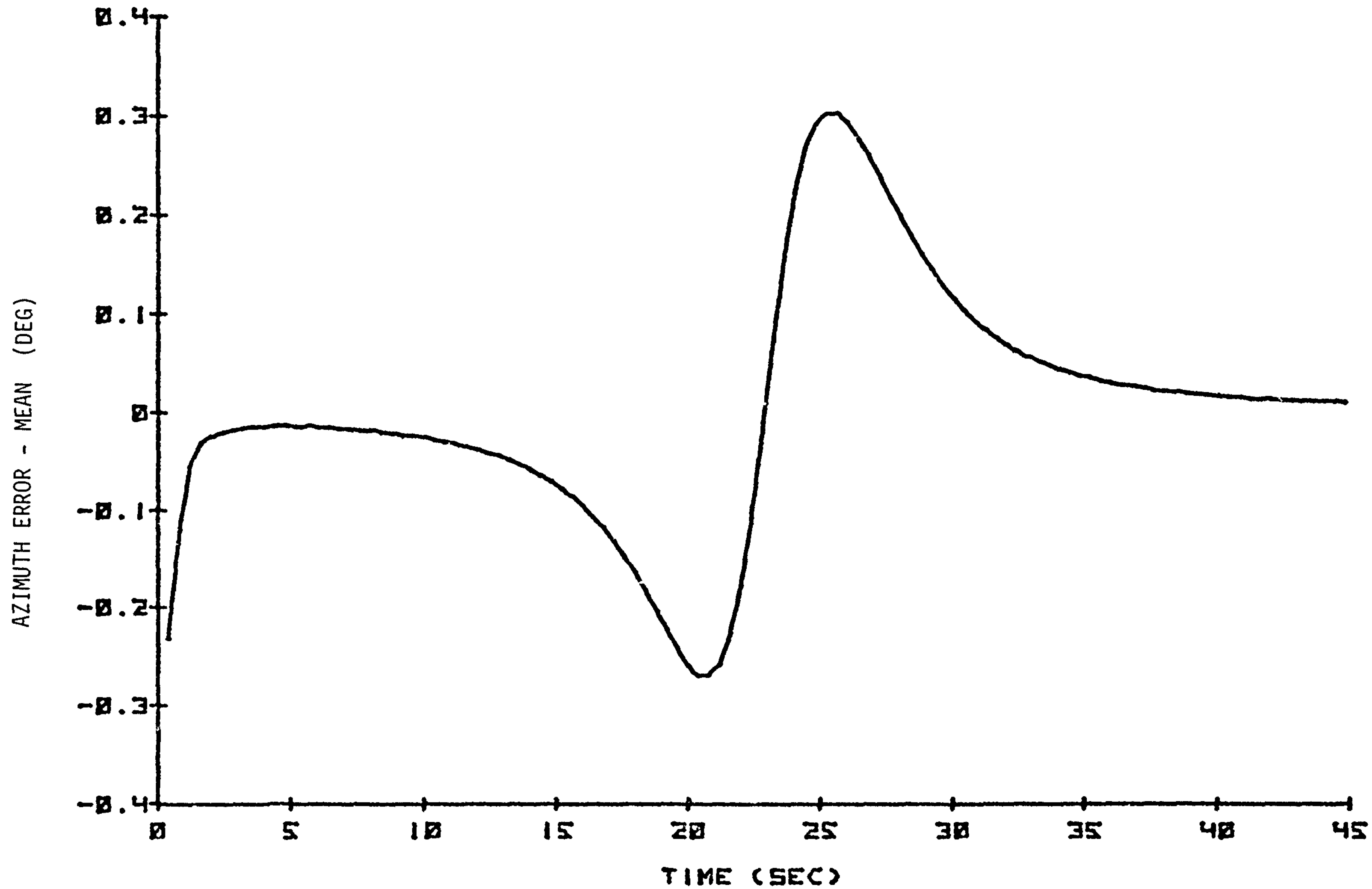


FIGURE 10A: OPTIMAL CONTROL MODEL PREDICTION (PHI FLYBY)

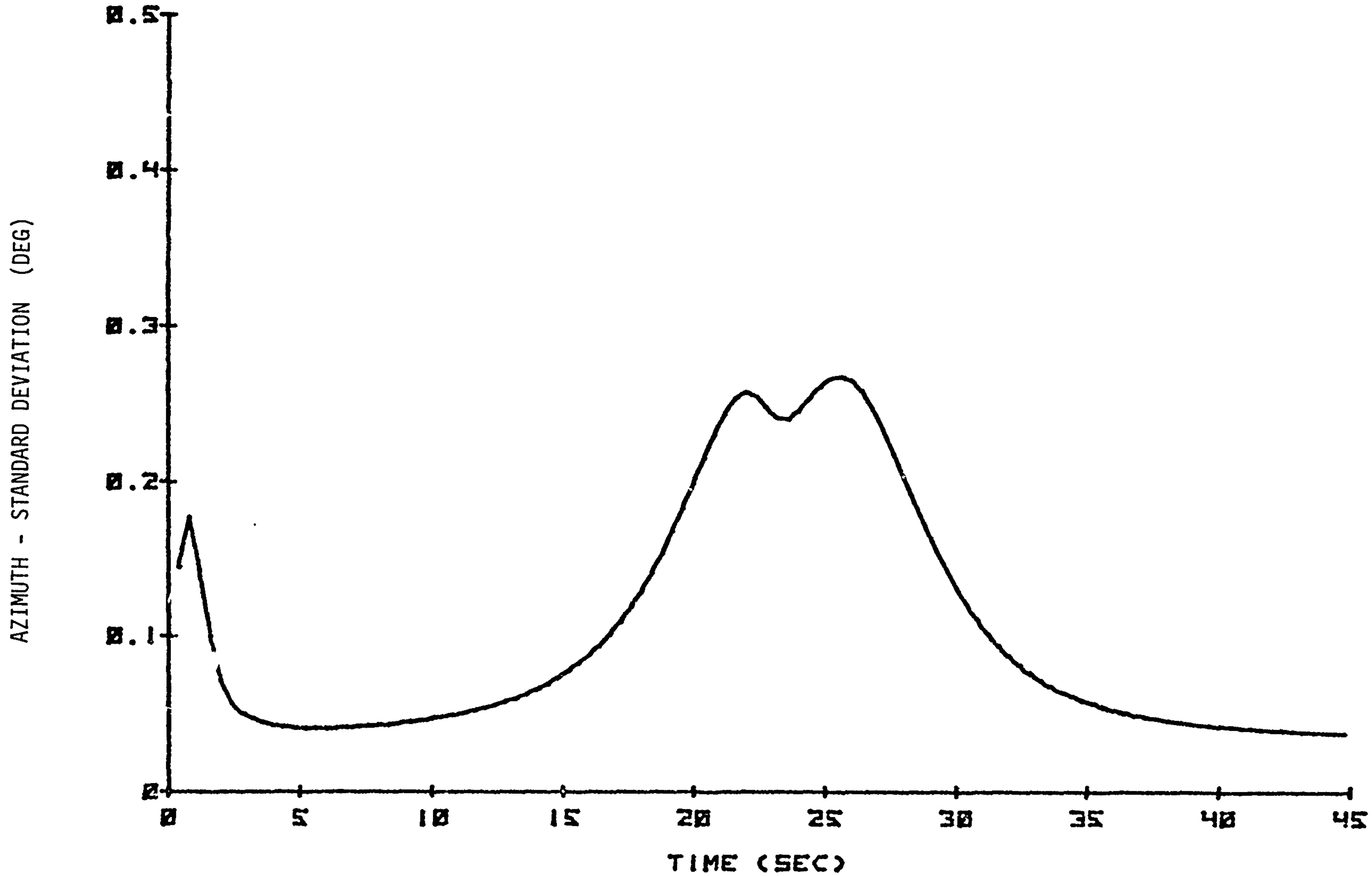


FIGURE 10B: OPTIMAL CONTROL MODEL PREDICTION (PHI FLYBY)

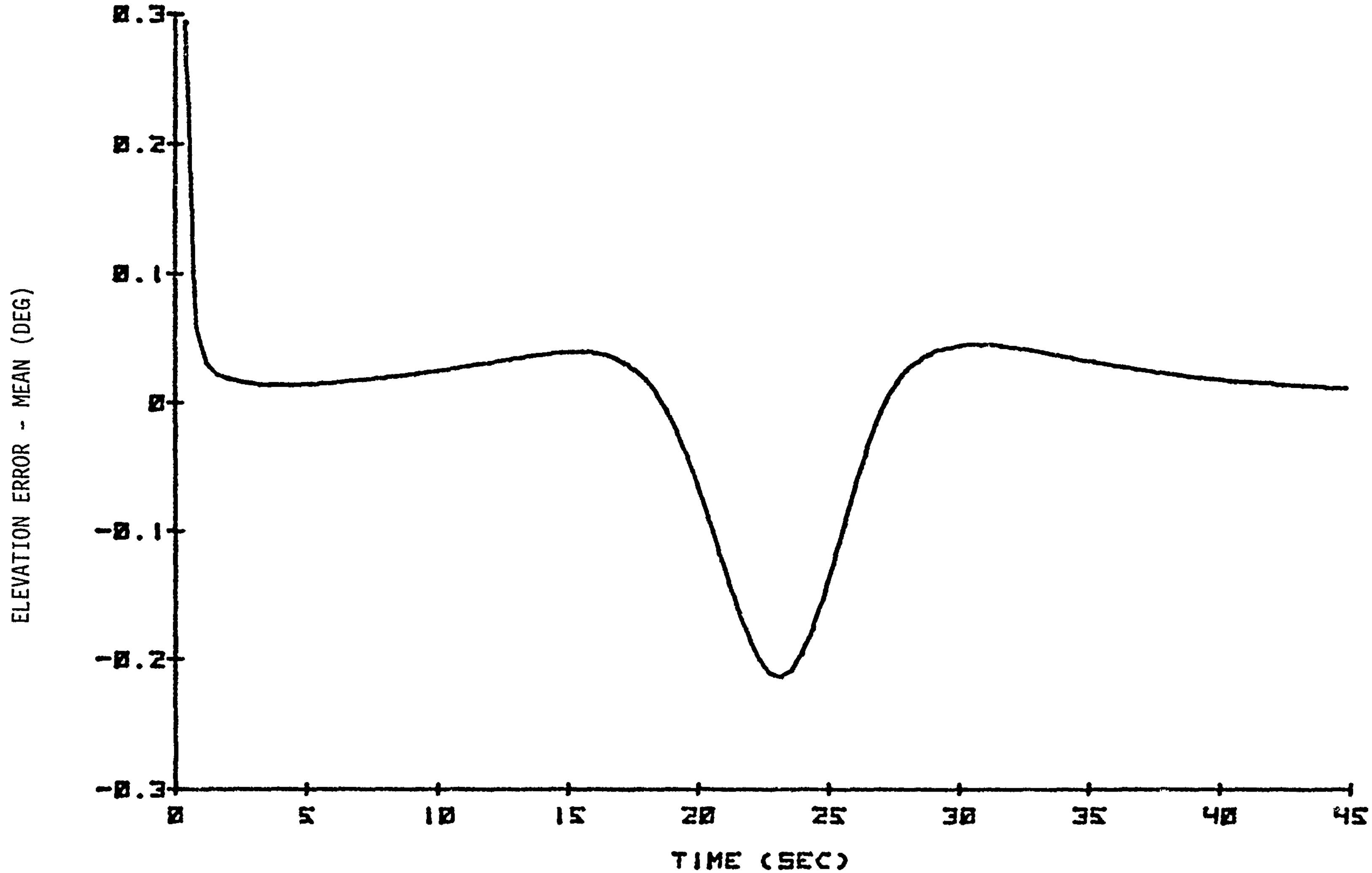


FIGURE 10C: OPTIMAL CONTROL MODEL PREDICTION (PHI FLYBY)

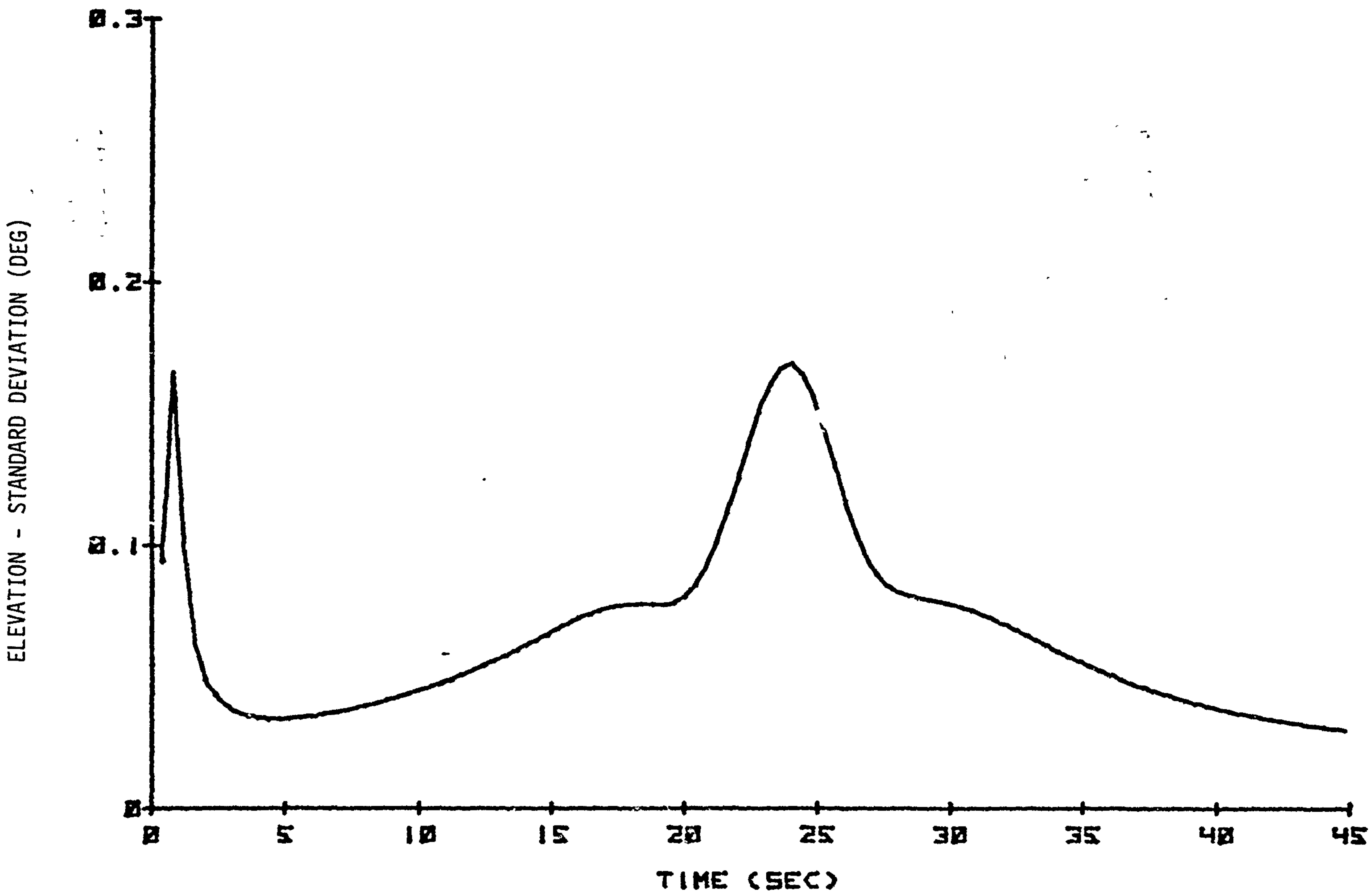


FIGURE 10D: OPTIMAL CONTROL MODEL PREDICTION (PHI FLYBY)

APPENDIX A

A Comparison of the Human Operator Models for AAA Systems

This appendix will give a general description and comparison to the following models:

1. McRuer Crossover Model (based on classical control theory)
2. Optimal Control Model (based on optimal control and estimation theory)
3. PID Structure Modified Optimal Control Model (simplified Optimal Control Model)
4. Observer Model (based on the observer theory)

1. McRuer Crossover Model

The structure of the McRuer crossover model is described in the frequency domain in Figure 11(a). $Y_C(s)$ is the transfer function of the controlled system and the crossover model is composed of a linear element $Y_H(s)$ and an random element (remnant). An interesting relationship of the open loop transfer function $Y_H(s) \cdot Y_C(s)$ was found by McRuer as follows:

$$Y_H\,(j\omega)\;Y_C(j\omega) \cong \frac{\omega_c e^{-j\omega\tau_e}}{j\omega}$$

around the region of crossover frequency. The two parameters ω_c and τ_e are crossover frequency and effective time delay respectively.

This model was developed using classical control theory, i.e., frequency domain analysis. It is a useful model of a human operator primarily applied to single-input single-output linear time-invariant systems with random or random-appearing inputs. The applications to multivariable systems or time-varying systems or deterministic input forcing functions are not straightforward.

2. Optimal Control Model

Optimal control model assumes that the human operator behaves in an optimal manner subject to his inherent limitations and to the requirements of the control

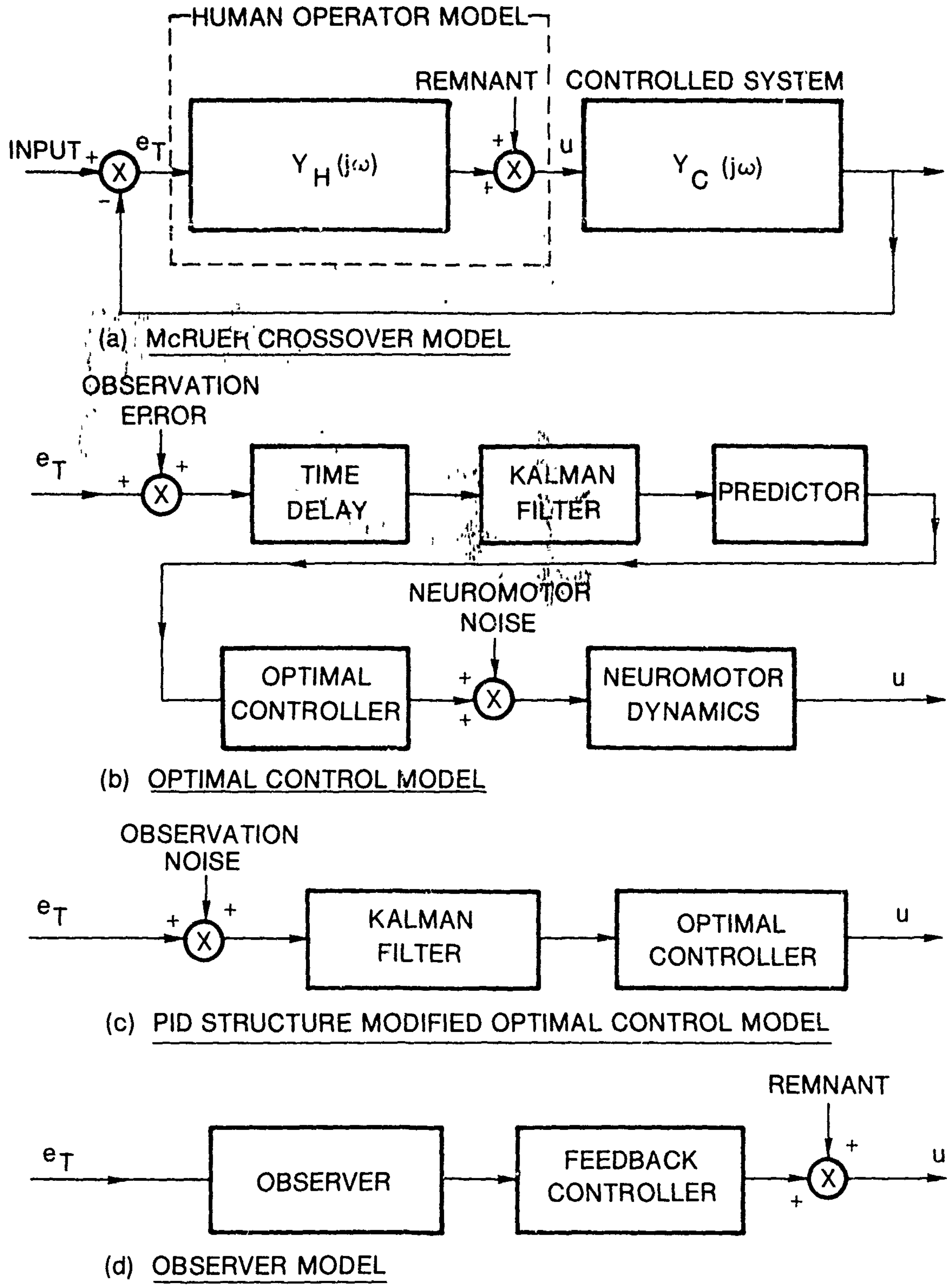


FIGURE 11. BLOCK DIAGRAMS OF HUMAN OPERATOR MODELS

task. Kalman filter is used in the model to obtain the optimal estimation of the state of the dynamic system. Then the optimal, linear, state variable feedback control is implemented by using the estimated state. The block diagram of the model is shown in Figure 11(b). Optimal control model is based on the state space techniques to describe the dynamic system and hence is convenient to be applied to multivariable systems or time-varying systems. So the optimal control model is a good gunner model for complex weapon systems. The problems of the optimal control model are summarized as follows:

a) The structure of the optimal control model is complicated. There are two Riccati equations (which are nonlinear matrix differential equations) involved in the model to be solved. Therefore the computer execution time to generate model predictions of the tracking errors using optimal control model is lengthy.

b) There is no systematic method to determine the parameters of the optimal control model. Usually it takes a long time to modify the cost function or model structure in order to obtain a set of parameter values to give satisfactory model predictions. Generally speaking, these parameters are determined through trial and error.

3. PID Structure Modified Optimal Control Model

The PID structure modified optimal control model is a simplified version of the optimal control model. Its structure is shown in Figure 11(c). Obviously, it is much simpler than the optimal control model. In addition, this model assumes thatthe human operator learns an input-output type of internal model relating the human's control u(t) to the displayed variable $e_T(t)$ (i.e., input to the human operator), without using any explicit representation of the actual system dynamics and forcing functior. The advantage of this idea is to have simple mathematical equations for the model. The disadvantage of this formulation is that it may be difficult to tune the parameters effectively to obtain accurate model predictions.

Another good idea in this model is the consideration of the lower order

internal model of the controlled plant. But this causes the divergence of the covariance matrix of the Kalman filter. Further study is necessary in order to use this idea successfully.

4. Observer Model

Two of the main design requirements in the development of the observer model are:

a) systematic determination of parameters of the human operator model,

b) simple structure of the model (this shortens execution time in the computer simulation of an AAA tracking task).

The Luenberger observer theory was applied to develop this model and its basic structure is shown in Figure 11(d) for comparison. In this figure, the observer plays the role of the Kalman filter and the feedback controller plays the role of the optimal controller. An observer generates an estimate of the state for a dynamic system. Its advantage is that no Riccati equation is required to compute the observer gain. (note that in the optimal control model to compute the optimal Kalman gain, a nonlinear matrix differential equation has to be solved.) Similarly there is no quadratic cost function involved in the formulation of observer model. Only a linear state-variable feedback control law is used in this model. Hence, the structure of the observer model is much simpler than that of the optimal control model.

In addition, the parameters of the observer model are not determined by trial and error tuning. The developed parameter identification program (i.e., least squares curve-fitting program) in Section III can identify the parameters systematically.

APPENDIX B

Development of Gauss-Newton Gradient Method

Consider a scalar function $J(\underline{a})$ of a vector parameter $\underline{a}$ and its first derivative $\partial J(\underline{a})/\partial \underline{a}$. Taking first order Taylor series expansion of $\partial J(\underline{a})/\partial \underline{a}$ with respect to a minimum $\underline{a}^*$ of $J(\underline{a})$, we have:

$$\frac{\partial J(\underline{a})}{\partial \underline{a}} = \frac{\partial J(\underline{a}^*)}{\partial \underline{a}} + \frac{\partial^2 J(\underline{a}^*)}{\partial \underline{a} \partial \underline{a}^T} \cdot [\underline{a} - \underline{a}^*] \tag{B1}$$

The first term of the right hand side of Eq. (B1) is zero because $\underline{a}^*$ is a minimum of $J(\underline{a})$. Now, if the second derivative of $J(\underline{a})$ with respect to $\underline{a}$ at $\underline{a}^*$ is not singular, then Eq. (B1) can be rewritten as follows:

$$\underline{a}^* = \underline{a} - [\frac{\partial^2 J(\underline{a}^*)}{\partial \underline{a} \partial \underline{a}^T}]^{-1} \cdot (\frac{\partial J(\underline{a})}{\partial \underline{a}}) \tag{B2}$$

Eq. (B2) can be used as a basis for an iterative relation to obtain the minimum of the function $J(\underline{a})$. The iterative relation is given by

$$\underline{a}_{i+1} = \underline{a}_i - \left[\frac{\partial^2 J(\underline{a}_i)}{\partial \underline{a} \partial \underline{a}^T}\right]^{-1} \frac{\partial J(\underline{a}_i)}{\partial \underline{a}} \tag{B3}$$

This is the Newton gradiant method. In order to satisfy the descent property, i.e., $J(\underline{a}_{i+1}) < J(\underline{a}_i)$, a one-dimensional search for ρ_i is performed to adjust the step size of increment in the iterative procedure. Newton method can now be written as:

$$\underline{a}_{i+1} = \underline{a}_i - \rho_i \left[\frac{\partial^2 J(\underline{a}_i)}{\partial \underline{a} \partial \underline{a}^T}\right]^{-1} \frac{\partial J(\underline{a}_i)}{\partial \underline{a}} \tag{B4}$$

The Gauss-Newton technique is an attempt to obtain convergence characteristics that are similar to the Newton method without calculating the second derivative of the criterion function with respect to $\underline{a}$. Consider a known function $f_1(x)$ and a second function $f(x,\underline{a})$ where $\underline{a}$ is a parameter vector of dimension p, to be determined in order to minimize the criterion function:

$$J(\underline{a}) = \int_{x_0}^{x_f} [f_1(x) - f(x, \underline{a})]^2 dx \tag{B5}$$

Where x_0, x_f are integral limits. Take a first order Taylor series expansion of $f(x, \underline{a})$ about a certain initial guess $\underline{a}_0$ and substitute into $J(\underline{a})$.

$$J(\underline{a}) = \int_{x_0}^{x_f} [f_1(x) - f(x, \underline{a}_0) - \frac{\partial f(x, \underline{a}_0)}{\partial \underline{a}} (\underline{a} - \underline{a}_0)]^2 dx \tag{B6}$$

For simplicity we will assume that $f_1(x)$ and $f(x, \underline{a})$ are acalar functions of x and $(x, \underline{a})$ respectively, and $\underline{a}$ is a p dimensional vector. Now the scalar criterion function $J(\underline{a})$ can be rewritten as:

$$J(\underline{a}) = \int_{x_0}^{xf} \{(f_1(x)-f(x,\underline{a}_0))^2 - 2(f_1(x)-f(x,\underline{a}_0)) \frac{\partial f(x,\underline{a}_0)}{\partial \underline{a}} (\underline{a} - \underline{a}_0) + \left[\frac{\partial f(x, \underline{a}_0)}{\partial \underline{a}} (\underline{a} - \underline{a}_0)\right]^2 \}dx \tag{B7}$$

Taking the partial derivative of $J(\underline{a})$ with respect to $\underline{a}$, we get

$$\frac{\partial J(\underline{a})}{\partial \underline{a}} = \int_{x_0}^{x_f} \left[-2(f_1(x) - f(x, \underline{a}_0)) \frac{\partial f(x, \underline{a}_0)}{\partial \underline{a}} + 2 \frac{\partial f(x, \underline{a}_0)}{\partial \underline{a}} (\underline{a}-\underline{a}_0) \frac{\partial f(x, \underline{a}_0)}{\partial \underline{a}} \right] dx \tag{B8}$$

Note that the gradient of the criterion function is zero at its minimum $\underline{a}^*$.

$$0 = \frac{\partial J(\underline{a}^*)}{\partial \underline{a}} = \int_{x_0}^{x_f} \left[-2 (f_1(x)-f(x, \underline{a}_0)) \frac{\partial f(x, \underline{a}_0)}{\partial \underline{a}} + 2 \frac{\partial f(x, \underline{a}_0)}{\partial \underline{a}} (a^*-\underline{a}) \frac{\partial f(x,\underline{a}_0)}{\partial \underline{a}} \right] dx \tag{B9}$$

So taking the transposition of the resulting equation gives us

$$\int_{x_0}^{x_f} \left(\frac{\partial f(x, \underline{a}_0)}{\partial \underline{a}}\right)^T (f_1(x) - f(x, \underline{a}_0))dx = \int_{x_0}^{x_f} \left(\frac{\partial f(x, \underline{a}_0)}{\partial \underline{a}}\right)^T \frac{\partial f(x, \underline{a}_0)}{\partial \underline{a}} (\underline{a}^*-\underline{a}_0) dx \tag{B10}$$

For the discrete time case, the analogous result is:

$$\sum_{k=1}^{K} \left(\frac{\partial f(x_k,\underline{a}_0)}{\partial \underline{a}}\right)^T (f_1(x_k) - f(x_k, \underline{a}_0)) = \sum_{k=1}^{K} \left(\frac{\partial f(x_k, \underline{a}_0)}{\partial \underline{a}}\right)^T \left(\frac{\partial f(x_k, \underline{a}_0)}{\partial \underline{a}}\right)(\underline{a}^*-\underline{a}_0) \tag{B11}$$

Hence,

$$\underline{a}^* = \underline{a}_0 + \left[\sum_{k=1}^{K} \left(\frac{\partial f(x_k, \underline{a}_0)}{\partial \underline{a}} \right)^T \frac{\partial f(x_k, \underline{a}_0)}{\partial \underline{a}} \right]^{-1} \sum_{k=1}^{K} \left(\frac{\partial f(x_k, \underline{a}_0)}{\partial \underline{a}} \right)^T \left[f_1(x_k) - f(x_k, \underline{a}_0) \right] \tag{B12}$$

The iterative relation is given by:

$$\underline{a}_{i+1} = \underline{a}_i - \left[\sum_{k=1}^{K} \left(\frac{\partial f(x_k, \underline{a}_i)}{\partial \underline{a}} \right)^T \left(\frac{\partial f(x_k, \underline{a}_i)}{\partial \underline{a}} \right) \right]^{-1} \left[\sum_{k=1}^{K} \left(\frac{\partial f(x_k, \underline{a}_i)}{\partial \underline{a}} \right)^T (f(x_k, \underline{a}_i) - f_1(x_k)) \right] \tag{B13}$$

In order to satisfy the descent property, i.e., $J(\underline{a}_{i+1}) < J(\underline{a}_i)$, a one dimensional search for ρ is required. Thus the modified Gauss-Newton iterative relationship is expressed as:

$$\underline{a}_{i+1} = \underline{a}_i - \rho \left[\sum_{k=1}^{K} \left(\frac{\partial f(x_k, \underline{a}_i)}{\partial \underline{a}} \right)^T \left(\frac{\partial f(x_k, \underline{a}_i)}{\partial \underline{a}} \right) \right]^{-1} \left[\sum_{k=1}^{K} \left(\frac{\partial f(x_k, \underline{a}_i)}{\partial \underline{a}} \right)^T (f(x_k, \underline{a}_i) - f_1(x_k)) \right] \tag{B14}$$

Let the second term of the right hand side of Eq. (B14) be denoted by $+ \rho \underline{D}_i$, then:

$$\underline{a}_{i+1} = \underline{a}_i + \rho \underline{D}_i$$

Initially $\rho = 1$. If the descent property is not satisfied than ρ is halved. This process continues until $J(\underline{a}_{i+1}) < J(\underline{a}_i)$ or a lower limit is reached. At each iteration ρ is reset and the criterion function is again checked.

Let

$$E_j = \frac{(\underline{D}_i)_j}{(\underline{a}_i)_j} \qquad \text{for } j = 1,2,\ldots.\ p.$$

where $(\underline{D}_i)_j$ and $(\underline{a}_i)_j$ are the j^{th} components of $\underline{D}_i$ and $\underline{a}_i$ respectively. The convergence of Eq. (B14) is established whenever

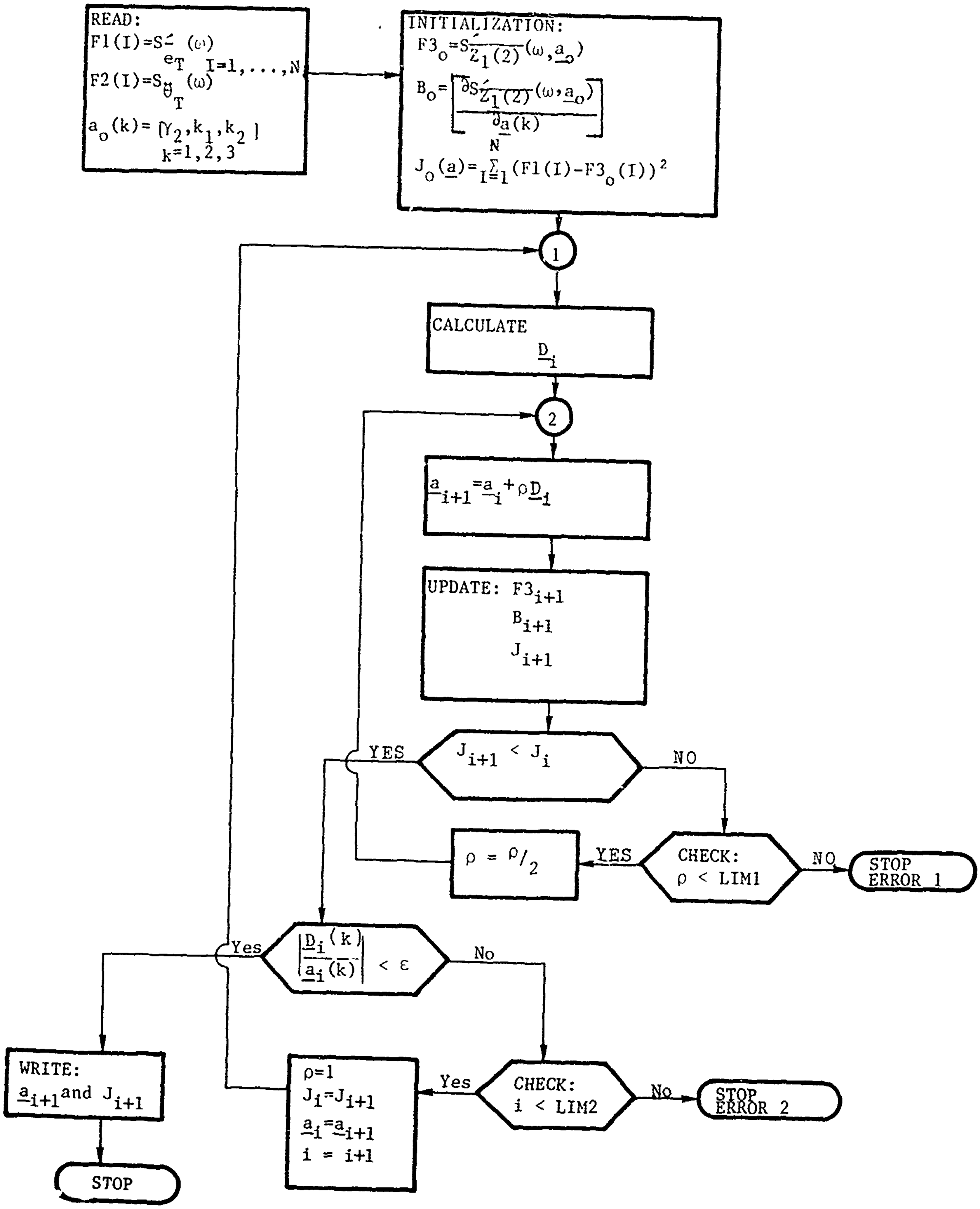
$$|E_j| < \epsilon \qquad \text{for } j = 1,2,\ldots.\ p.$$

where ϵ is a preassigned small positive number. Otherwise, further iteration is required.

Following is a flowchart for a computer implementation of the modified Gauss-Newton method.

APPENDIX C - PARAMETER IDENTIFICATION PROGRAM

A. FLOW CHART OF PARAMETER IDENTIFICATION FOR PARAMETERS ASSOCIATED WITH MEAN TRACKING ERROR EQUATION



COMMENTS: EXECUTION OF PROGRAM FIT1

1. COMPILE PROGRAM

2. INPUT: ATTACH TAPE1 AND TAPE2 where

   TAPE1 CONTAINS EMPIRICAL STANDARD DEVIATION OF TRACKING ERROR

   TAPE2 HAS PARTIAL DERIVATIVE INFORMATION (described later in SDCON)

3. ATTACH D.L. KLEINMAN [13] LIBRARY

   SUBROUTINE - GMINV, VMATI, VSCALE

4. RUN

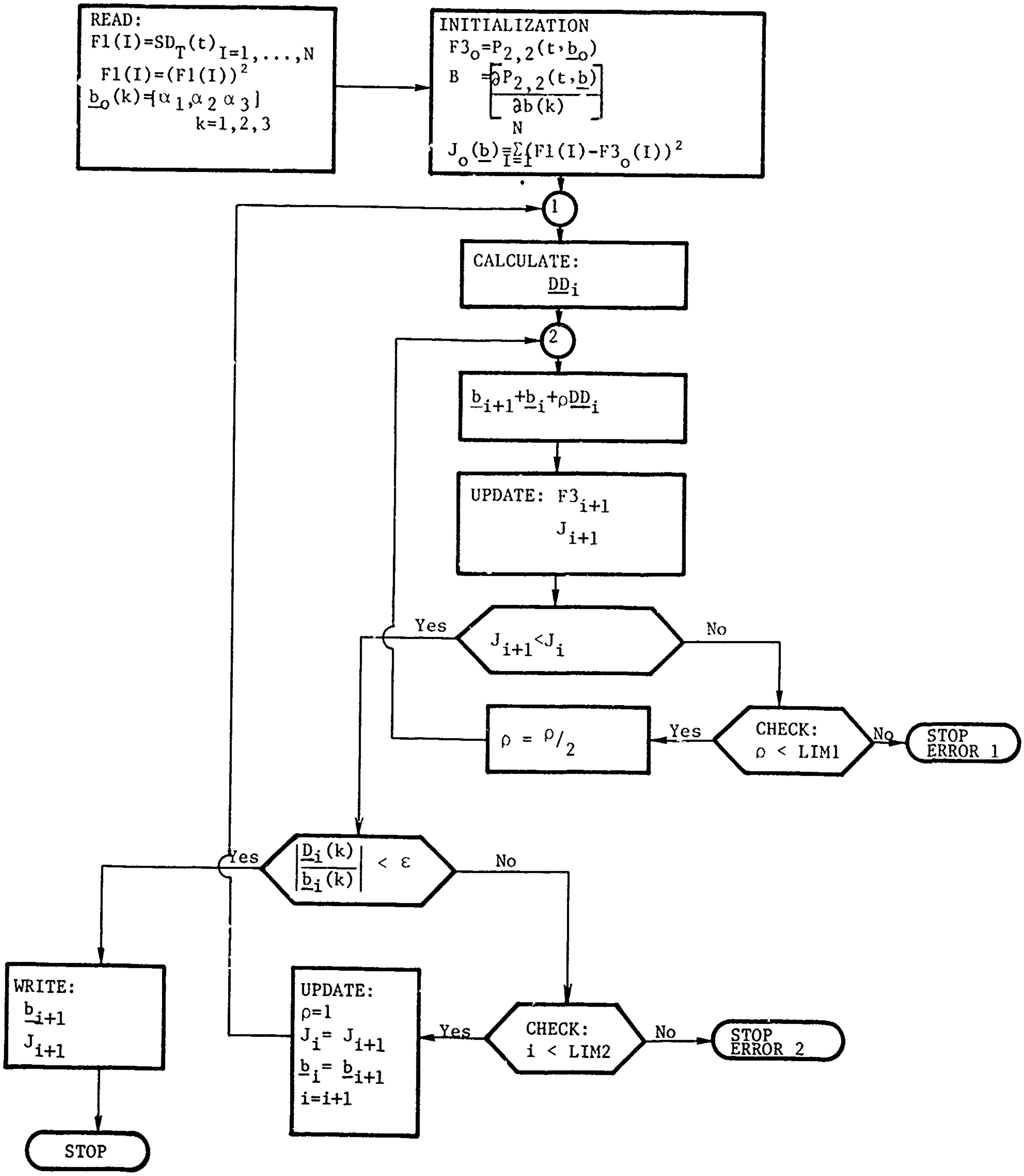
5. OUTPUT: PARAMETERS VALUES FOR $[\alpha_1, \alpha_2, \alpha_3]$

```
      PROGRAM FIT(INPUT,OUTPUT,TAPE1,TAPE2)
1     PRINT*,50HTYPE 1 TO GO OR TYPE 0 TO STOP
      READ*,IFG
      IF(IFG.EQ.0)STOP
      REWIND 1
      REWIND 2
      CALL INIT
      GO TO 1
      END
      SUBROUTINE INIT
C     INITIALIZATION ROUTINE
C     READ IN DATA TO BE FITTED
C     MAKE INITIAL GUESS
      COMMON/VAR/LIM1,LIM2,EE,H,N,DEL
      COMMON/MAT/F1(1125),F2(1125),F3(1125),A(3),AA(3),B(1125,3),D(3)
      DATA N,H,IP/1125,.04,3/
      PRINT*,30H TYPE IN LIM1,LIM2,EE
      READ*,LIM1,LIM2,EE
      PRINT*,3H N=,N,5HLIM1=,LIM1,5HLIM2=,LIM2,3HEE=,EE,3HIP=,IP
      H=.04
      DO 10 I=1,N
      READ(1,3)Z,Z1,F1(I),Z2
      F1(I)=F1(I)*F1(I)
3     FORMAT(4G12.5)
      READ(2,4)F2(I),(B(I,J),J=1,IP)
4     FORMAT(4G12.5)
10    CONTINUE
      PRINT*,50HTYPE IN INITIAL GUESS--ALP 1,ALP 2,ALP 3
      READ*,AA
      PRINT*,10H 1ST GUESS,AA
      DEL=1
      CALL COEF(AA,IP)
      CALL JMIN(RJ2)
      PRINT*,10H JMIN(1)= ,RJ2
      CALL LOOP(RJ2,IP)
      END
      SUBROUTINE JMIN(RJ)
C     COMPUTE RJ=CRITERION FUNCTION
      COMMON/VAR/LIM1,LIM2,EE,H,N,DEL
      COMMON/MAT/F1(1125),F2(1125),F3(1125),A(3),AA(3),B(1125,3),D(3)
      SUM=0.
      DO 10 I=1,N
10    SUM=SUM+(F1(I)-F3(I))**2
      RJ=SUM
      END
      SUBROUTINE DD(IP)
C     CALCULATE D MATRIX FROM Q AND R
      DIMENSION R(3),Q(3,3),W1(3,3)
      COMMON/VAR/LIM1,LIM2,EE,H,N,DEL
      COMMON/MAT/F1(1125),F2(1125),F3(1125),A(3),AA(3),B(1125,3),D(3)
      COMMON/MAIN1/NDIM,NDIM1,COM1(5,5)/INOU/KIN,KOUT
      NDIM=IP$NDIM1=IP+1
      KIN=KOUT=6
      DO 10 I=1,IP
      R(I)=0.
      DO 10 J=1,IP
10    Q(I,J)=0.
      DO 35 K=1,N
```

Note: Matrix *B* is computed in a seperate routine SDCON (listing follows) since B remains constant throughout the iteration process.

```
      SN=F3(K)-F1(K)
      DC 30 I=1,IP
      DO 25 J=1,IP
 25   Q(I,J)=Q(I,J)+B(K,I)*B(K,J)
 30   R(I)=R(I)+B(K,I)*SN
 35   CONTINUE
      CALL GMINV(IP,IP,Q,W1,MR,0)
      CALL VMAT1(W1,R,IP,IP,D)
      CALL VSCALE(D,D,IP,-1.)
      END
      SUBROUTINE LOOP(RJ2,IP)
C     ITERATION PROCESS
C     COMPUTE A(I+1)=A(I)+DEL*D(I)
C     DETERMINE WHEN A(I+1) IS ACCEPTABLE
      DIMENSION DDD(4)
      COMMON/VAR/LIM1,LIM2,EP,H,N,DEL
      COMMON/MAT/F1(1125),F2(1125),F3(1125),A(3),AA(3),B(1125,3),D(3)
      NCT=MCT=0.
 1    CALL DD(IP)
 2    DO 100 I=1,IP
      D(I)=D(I)*DEL
 100  A(I)=AA(I)+D(I)
      CALL COEF(1)
 5    CALL JMIN(RJ1)
 6    IF(RJ1.LT.RJ2)GO TO 30
      DEL=DEL/2.
      NCT=NCT+1
      IF(NCT.LE.LIM1)GO TO 2
      PRINT*,7HERROR 1,3H A=,A,7H NCT=,NCT,5H MCT=,MCT,RJ1,RJ2
      STOP
 30   DO 35 I=1,IP
      DDD(I)=ABS(D(I)/AA(I))
 35   IF(DDD(I).GT.EP)GO TO 20
      GO TO 40
 20   MCT=MCT+1
      IF(MCT.LE.LIM2)GO TO 9
      PRINT*,7HERROR 2,3H A=,A,7H NCT=,NCT,5H MCT=,MCT,RJ1,RJ2
      STOP
 9    RJ2=RJ1
      NCT=0
      DEL=1.
      DO 25 I=1,IP
 25   AA(I)=A(I)
      GO TO 1
 40   PRINT*,4H  A=,A,6HNCT=,NCT,6HMCT=,MCT
      PRINT*,3HAA=,AA
      PRINT*,3H D=,D
      PRINT*,5HJMIN=,RJ1
      END
      SUBROUTINE COEF(W,IP)
C     COMPUTE FUNCTION F3(W,A) GIVEN PARAMETERS A
C     PARTIAL DERIVATIVES OF F3(W,A) WRT A ARE IN MATRIX B
      DIMENSION W(IP)
      COMMON/VAR/LIM1,LIM2,EP,H,N,DEL
      COMMON/MAT/F1(1125),F2(1125),F3(1125),A(3),AA(3),B(1125,3),D(3)
      DO 10 I=1,N
      F3(I)=F2(I)+B(I,1)*W(1)+B(I,2)*W(2)+B(I,3)*W(3)
 10   CONTINUE
      END
```

B. FLOW CHART OF PARAMETER IDENTIFICATION FOR PARAMETERS ASSOCIATED WITH STANDARD DEVIATION OF TRACKING ERROR EQUATION



COMMENTS: EXECUTION OF PROGRAM FIT

1. COMPILE PROGRAM

2. INPUT: ATTACH, TAPE1 and TAPE2 where

   TAPE1 CONTAINS PSD OF EMPIRICAL MEAN TRACKING ERROR

   TAPE2 CONTAINS PSD OF $\ddot{\theta}_T$ FOR GIVEN TRAJECTORY

3. ATTACH D.L. KLEINMAN [13] LIBRARY

   SUBROUTINE USED - GMINV, VMAT1, VSCALE

4. RUN

5. OUTPUT: PARAMETER VALUES FOR [γ_2, k_1, k_2]

```
      PROGRAM FIT(INPUT,OUTPUT,TAPE1,TAPE2,TAPE6)
1     PRINT*,50HTYPE 1 TO GO OR TYPE 0 TO STOP
      READ*,IFG
      IF(IFG.EQ.0)STOP
      REWIND 1
      REWIND 2
      CALL INIT
      GO TO 1
      END
      SUBROUTINE INIT
C     INITIALIZATION ROUTINE
C     READ IN DATA TO BE FITTED
C     MAKE INITIAL GUESS
      COMMON/VAR/LIM1,LIM2,EE,IP,H,N,DEL
      COMMON/MAT/F1(513),F2(513),F3(513),A(3),AA(3),B(513,3),D(3)
      DATA N,H,IP/513,.02**1,3/
      PRINT*,30H TYPE IN LIM1,LIM2,EE
      READ*,LIM1,LIM2,EE
      PRINT*,3H N=,N,2HH=,H,5HLIM1=,LIM1,5HLIM2=,LIM2,3HEE=,EE,3HIP=,IP
      DO 10 I=1,N
      READ(1,3)Z,F1(I)
      READ(2,4)Z,F2(I)
4     FORMAT(1X,2G12.4)
3     FORMAT(1X,2G12.5)
10    CONTINUE
      PRINT*,30H PRINT FIRST GUESS
      READ*,AA
      PRINT*,10H 1ST GUESS,AA
      DEL=1
      CALL COEF(AA)
      CALL JMIN(RJ2)
      CALL LOOP(RJ2)
      FREQ=0.
      DO 20 I=1,N
      WRITE(6,25)FREQ,F3(I)
20    FREQ=FREQ+H
25    FORMAT(2G12.5)
      END
      SUBROUTINE JMIN(RJ)
C     COMPUTE RJ=CRITERION FUNCTION
      COMMON/VAR/LIM1,LIM2,EE,IP,H,N,DEL
      COMMON/MAT/F1(513),F2(513),F3(513),A(3),AA(3),B(513,3),D(3)
      SUM=0.
      DO 10 I=1,N
10    SUM=SUM+(F1(I)-F3(I))**2
      RJ=SUM
      END
      SUBROUTINE DD
C     CALCULATE D MATRIX FROM Q AND R
      DIMENSION R(3),Q(3,3),W1(3,3)
      COMMON/VAR/LIM1,LIM2,EE,IP,H,N,DEL
      COMMON/MAT/F1(513),F2(513),F3(513),A(3),AA(3),B(513,3),D(3)
      COMMON/MAI1/NDIM,NDIM1,COM1(3,3)/INOU/KIN,KOUT
      NDIM=3$NDIM1=4
      KIN=KOUT=6
      DO 10 I=1,IP
      R(I)=0.
      DO 10 J=1,IP
```

```
10    Q(I,J)=0.
      DO 35 K=1,N
      SN=F3(K)-F1(K)
      DO 30 I=1,IP
      DO 25 J=1,IP
25    Q(I,J)=Q(I,J)+B(K,I)*B(K,J)
30    R(I)=R(I)+B(K,I)*SN
35    CONTINUE
      CALL GMINV(IP,IP,Q,W1,MR,0)
      CALL VMAT1(W1,R,IP,IP,D)
      CALL VSCALE(D,D,IP,-1.)
      END
      SUBROUTINE LOOP(RJ2)
C     ITERATION PROCESS
C     COMPUTE A(I+1)=A(I)+DEL*D(I)
C     DETERMINE WHEN A(I+1) IS ACCEPTABLE
      DIMENSION DDD(3)
      COMMON/VAR/LIM1,LIM2,FT,IP,H,N,DEL
      COMMON/MAT/F1(513),F2(513),F3(513),A(3),AA(3),B(513,3),D(3)
      NCT=MCT=0.
1     CALL DD
2     DO 100 I=1,IP
      D(I)=D(I)*DEL
100   A(I)=AA(I)+D(I)
      CALL COEF(1)
5     CALL JMIN(RJ1)
6     IF(RJ1.LT.RJ2)GO TO 30
      DEL=DEL/2.
      NCT=NCT+1
      IF(NCT.LE.LIM1)GO TO 2
      PRINT*,7HERROR 1,3H A=,A,5H NCT=,NCT,5H MCT=,MCT,RJ1,RJ2
      STOP
30    DO 35 I=1,IP
      DDD(I)=ABS(D(I)/AA(I))
35    IF(DDD(I).GT.EE)GO TO 20
      GO TO 40
20    MCT=MCT+1
      IF(MCT.LE.LIM2)GO TO 9
      PRINT*,7HERROR 2,3H A=,A,5H NCT=,NCT,5H MCT=,MCT,RJ1,RJ2
      STOP
9     RJ2=RJ1
      NCT=0
      DEL=1.
      DO 25 I=1,IP
25    AA(I)=A(I)
      GO TO 1
40    PRINT45,(A(I),I=1,IP),NCT,MCT
      PRINT*,3HAA=,AA
      PRINT*,3H D=,D
      PRINT*,5HJMIN=,RJ1
45    FORMAT(1X,3G12.5,2X,2I5)
```

```
      END
      SUBROUTINE COFF(W)
C     COMPUTE FUNCTION F3(W,A) GIVEN PARAMETERS A
C     EXPRESS PARTIAL DERIVATIVES OF F3(W,A) WRT A IN MATRIX B
      DIMENSION W(3)
      COMMON/VAR/LIM1,LIM2,IE,IP,H,N,DEL
      COMMON/MAT/F1(513),F2(513),F3(513),A(3),AA(3),B(513,3),D(3)
      A1(P,Q,R)=P-R
      A2(P,Q,R)=R*R+P*P-2.*Q
      A3(P,Q,R)=Q*Q+P*P*R*R-2.*P*R*Q
      A4(P,Q,R)=P*Q
      PI=2.*3.14159
      WW=FREQ=0.
      DO 10 I=1,N
      TN1=(A1(W(1),W(2),W(3))**2)
      SN=WW*WW*(WW*WW*A2(W(1),W(2),W(3))+A3(W(1),W(2),W(3)))
      SN=(WW**6)+SN+(A4(W(1),W(2),W(3))**2)
      TN=(A4(W(1),W(2),W(3))**2)*(WW*WW+TN1)
      F3(I)=TN*F2(I)/(TN1*SN)
      B(I,1)=WW*WW*(WW*WW*W(1)+W(1)*(W(3)*W(3)-2.*W(2)))
      B(I,1)=TN1*(B(I,1)+A4(W(1),W(2),W(3))*W(2))+A1(W(1),W(2),W(3))*SN(
      SN1=A1(W(1),W(2),W(3))*A4(W(1),W(2),W(3))+W(2)*(WW*WW+TN1)
      SN1=(SN1*TN1*A4(W(1),W(2),W(3))*SN-TN*B(I,1))*2.*F2(I)
      B(I,1)=SN1/(TN1*TN1*SN*SN)
      B(I,2)=(WW*WW*(-WW*WW+W(2)-W(1)*W(1))+A4(W(1),W(2),W(3))*W(1))*TN(
      B(I,2)=(A4(W(1),W(2),W(3))*W(1)*(WW*WW+TN1)*SN-B(I,2))*2.*F2(I)
      B(I,2)=B(I,2)/(TN1*SN*SN)
      B(I,3)=A1(W(1),W(2),W(3))*WW*WW*(W(3)*WW*WW+W(1)*W(1)*W(3))-SN
      B(I,3)=-(A4(W(1),W(2),W(3))**2)*TN1*SN-TN*B(I,3)
      B(I,3)=2.*F2(I)*B(I,3)/((A1(W(1),W(2),W(3))**3)*SN*SN)
      FREQ=FREQ+H
      WW=PI*FREQ
10    CONTINUE
      END
```

COMMENTS: EXECUTION OF PROGRAM SDCON

1. COMPILE PROGRAM

2. INPUT: ATTACH, TAPE1 which has TRAJECTORY INFORMATION

   θ_T, $\dot{\theta}_T$, $\ddot{\theta}_T$, ϕ_T, $\dot{\phi}_T$, $\ddot{\phi}_T$

3. ATTACH INTERNATIONAL MATHEMATICAL & STATISTICAL LIBRARY (IMSL)

   SUBROUTINE USED - VCONVO

4. RUN

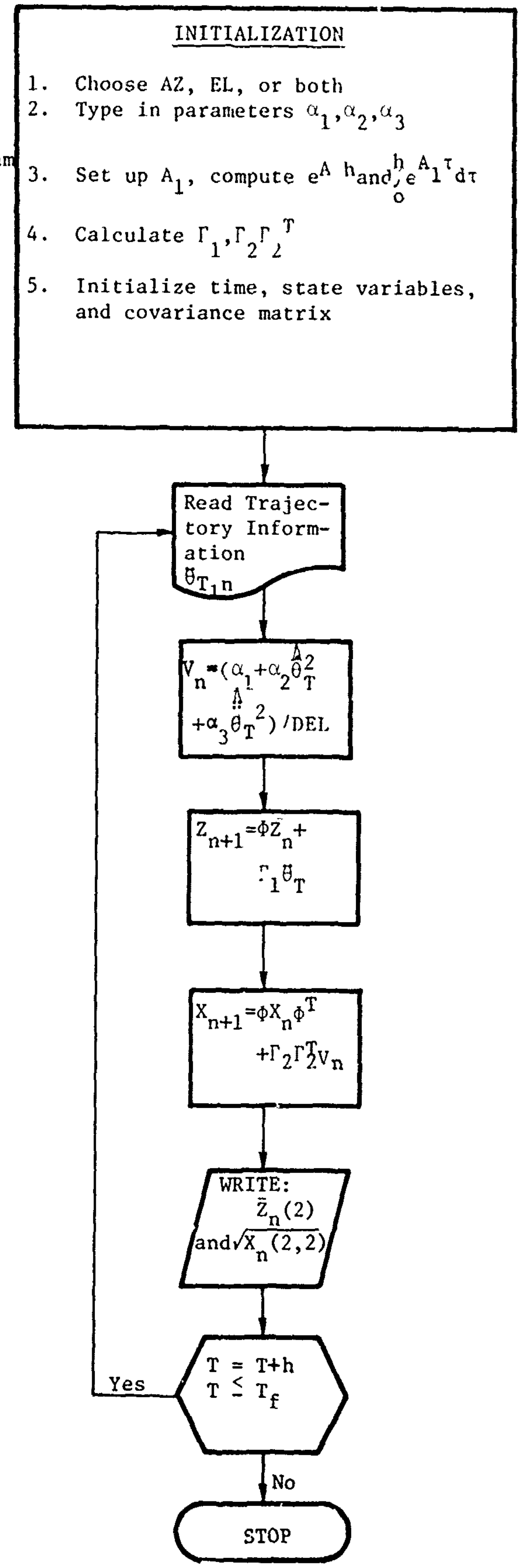
5. OUTPUT: TAPE2

   PARTIAL DERIVATIVES OF P(2,2) WITH RESPECT TO (α_1, α_2, α_3).

```
      PROGRAM SCDON(INPUT,OUTPUT,TAPE1,TAPE2)
      DIMENSION A(4096),A1(1024),B(2048),IWK(12),C(9,2),F2(1024),D(1024)
      DATA C/.1,-1.77,-1.3,4.83,1.64,-1.2,.26,130.,5.76,.35,-1.877,-1.88,
     1 4.63,.05,-1.33,.4,188.,1.,1.14/
      N=1024$DEL=.04
C     TAPE1 IS INPUT TAPE WITH TRAJECTORY INFO IN DEGREES
C     TAPE2 IS OUTPUT TAPE2 WITH PARTIAL DERIVATIVES OF P(2,2)
C     IC=1 FOR AZIMUTH AND IC=2 FOR ELEVATION
      READ*,IC
C     A(I)=Q(2,3)      A1(I)=Q(2,4)
C     V=P1+P2*(OT DOT)*(OT DOT)+P3(OT DOT)*(OT DOT)
C     F2(I)=Q(I,I)*Q(I,I)
      DO 10 I=1,N
      T=(I-1)*DEL
      A(I)=-EXP(C(3,IC)*T)*(COS(C(4,IC)*T)+C(5,IC)*SIN(C(4,IC)*T))
      A(I)=C(1,IC)*(EXP(C(2,IC)*T)-A(I))
      A1(I)=-EXP(C(3,IC)*T)*(COS(C(4,IC)*T)+C(7,IC)*SIN(C(4,IC)*T))
      F2(I)=A(I)*A(I)+A1(I)*A1(I)+(EXP(C(2,IC)*T))**2
      A(I)=A1(I)=625.*A(I)*A(I)-C(8,IC)*A(I)*A1(I)+C(9,IC)*A1(I)*A1(I)
      READ(1,5)CT,B(I),D(I),E,ED,EDD
      IF(IC.EQ.1)GO TO 6
      B(I)=ED
      D(I)=EDD
6     B(I)=B(I)*B(I)
      D(I)=D(I)*D(I)
10    CONTINUE
5     FORMAT(6G12.4)
C     PARTIAL OF P(2,2) WRT P2
      CALL VCONVD(A,B,N,N,IWK)
      DO 100 I=1,N
      B(I)=D(I)
      D(I)=A(I)
100   A(I)=A1(I)
C     PARTIAL OF P(2,2) WRT P3
      CALL VCONVD(A,B,N,N,IWK)
      DO 20 I=1,N
      AA=A1(I)
      A1(I)=A(I)
      A(I)=AA
20    B(I)=1.
C     PARTIAL OF P(2,2) WRT P1
      CALL VCONVD(A,B,N,N,IWK)
      DO 30 I=1,N
30    WRITE(2,4)F2(I),A(I),D(I),A1(I)
4     FORMAT(4G12.5)
      END
```

APPENDIX D

A. Flow Chart Of Simulation Program



COMMENTS: EXECUTION OF PROGRAM OBS

1. COMPILE ROUTINE

2. INPUT: ATTACH, TAPE2

   where TAPE2 HAS THE FOLLOWING CHANNELS:

   θ_T, $\dot{\theta}_T$, $\ddot{\theta}_T$, ϕ_T, $\dot{\phi}_T$, $\ddot{\phi}_T$ FOR A GIVEN TRAJECTORY

3. ATTACH D.L. KLEINMAN [13] LIBRARY

   SUBROUTINE USED - DSCRT

4. RUN

5. OUTPUT: TIME, MEAN TRACKING ERROR, STANDARD DEVIATION

```
      PROGRAM OBS(INPUT,OUTPUT,TAPE2)
      DIMENSION A(4,4),Z(4),X(4,4),R1(4),W1(4,4),W2(4,4),
     1 P(5,2)
      COMMON/MAIN1/N,N2
C     TIME: 0. - 45.  TIME STEP=.04
C     P = LAMDA1, LAMDA2, K1, K2, OT DOT
      DATA DEL,N,TEND/.04,4,45./
      DATA P/-1.,-1.77,25.,2.6,0.,-1.,-1.877,25.,3.76,0./
      N1=N*N
      N2=N+1
      L1=1
      L2=2
      PRINT*,50H TYPE 1 FOR AZ   2 FOR EL   3 FOR BOTH
      READ*,IFG
      IF(IFG.EQ.1)L2=1
      IF(IFG.EQ.2)L1=2
C     IC=1 FOR AZIMUTH AND IC=2 FOR ELEVATION
      DO 500 IC=L1,L2
      REWIND 2
C     P1,P2,P3 ARE COEFFICIENTS ASSOCICATED WITH NOISE COVARIANCE V.
      PRINT*,20H TYPE IN P1 P2 P3
      READ*,P1,P2,P3
C     INITIALIZE VECTORS AND MATRICES
      DO 10 I=1,N1
      A(I)=0.
10    X(I)=0.
      A(2)=1+P(1,IC)
      A(6)=P(2,IC)
      A(10)=-P(1,IC)
      A(14)=-P(2,IC)
      A(15)=-P(3,IC)
      A(12)=1.
      A(16)=-P(4,IC)
      CALL DSCRT(N,A,DEL,W1,W2,10)
C     W1=TRANSITION MATRIX   W2=INTEGRAL OF TRANSITION MATRIX
      JJ=1
      DO 65 J=5,8
      J1=J+8
      R1(JJ)=-W2(J)-W2(J1)
65    JJ=JJ+1
      DO 60 I=1,N
      Z(I)=0.
      DO 60 J=1,N
60    A(I,J)=R1(I)*R1(J)
      DO 20 I=1,N
      II=I+2*N
20    R1(I)=W2(I)+W2(II)*(1-P(5,IC))
      T=0.
C     READ IN TRAJECTORY INFO FOR AZ AND EL IN DEGREES
1     READ(2,3)(P(I,1),I=1,3),(P(I,2),I=1,3)
3     FORMAT(6G12.4)
C     COMPUTE ESTIMATED OT DOT AND OT DDT
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      P4=Z(3)-Z(1)
      P5=(P4-PP4)/DEL
      IF(T.EQ.0.)P5=P(3,IC)
C     V=NOISE COVARIANCE
      V=(P1*P4*P4+P2*P5*P5+P3)/DEL
      PP4=P4
C     COMPUTE MEAN STATE EQUATION
      DO 25 I=1,N
      W2(I)=0.
      II=1
      DO 15 J=I,N1,N
      W2(I)=W2(I)+W1(J)*Z(II)
15    II=II+1
25    CONTINUE
      DO 35 I=1,N
      Z(I)=W2(I)+R1(I)*P(3,IC)
35    CONTINUE
C     COMPUTE COVARIANCE MATRIX OF STATE EQUATION
      CALL MULT(W1,X,N,N1,W2)
      DO 40 I=1,N1
40    X(I)=A(I)*V+W2(I)
      SD=SQRT(X(2,2))
      T=T+DEL
C     MEAN TRACKING ERROR IS Z(2)
C     VARIANCE OF TRACKING ERROR IS X(2,2)
C     PRINT OUTPUT AT EVERY SECOND
      LK=(T+.001)/DEL
      IF(MOD(LK,25).EQ.0)PRINT75,T,Z(2),SD
75    FORMAT(3X,3G12.4)
      IF(T.GE.TEND)GO TO 500
      GO TO 1
500   CONTINUE
      END
      SUBROUTINE MULT(E,F,L,L1,H)
      DIMENSION E(L1),F(L1),G(16),H(L1)
      DO 10 I=1,L
      II=1
      DO 10 K=1,L
      TEMP=0.
      DO 5 J=I,L1,L
      TEMP=TEMP+E(J)*F(II)
5     II=II+1
      KK=(K-1)*L+I
10    G(KK)=TEMP
      DO 20 I=1,L
      DO 20 K=I,L
      TEMP=0.
      II=K
      DO 15 J=I,L1,L
      TEMP=TEMP+G(J)*E(II)
15    II=II+L
      KK=(K-1)*L+I
20    H(KK)=TEMP
      L2=L-1
      DO 30 I=1,L2
      L3=I+1
      DO 30 J=L3,L
      K1=(I-1)*L+J
      K2=(J-1)*L+I
30    H(K1)=H(K2)
      END
```

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